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$\mathbb{A}^1$-fibrations and $\mathbb{A}^1_*$-fibrations on affine threefolds

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We use $\mathbb{A}^1$-fibrations (or the quotient morphisms by G_a-actions) and $\mathbb{A}^1_*$-fibrations (or the quotient morphisms by G_m-actions) to look into the structure of higher-dimensional affine algebraic varieties. Here the symbol $\mathbb{A}^1_*$ signifies the affine line $\mathbb{A}^1$ minus one point.

$\mathbb{A}^1$-fibrations are factored by the quotient morphisms by G_a-actions, hence docile except the problem of finite generation of the invariant subring under G_a-actions (Hilbert's fourteenth problem). But the $\mathbb{A}^1_*$-fibrations are not necessarily factored by the quotient morphisms by G_m-actions. We state a sufficient condition in terms of singular fibers of the $\mathbb{A}^1_*$-fibrations.

We consider $\mathbb{A}^1_*$-fibrations on $\mathbb{Q}$-acyclic threefolds and contractible threefolds. One result concerns a construction of more threefolds of the same kind by affine modifications due to Kaliman and Zaidenberg. These objects considered in dimension three seem to be more prosperous than in the case of surfaces. A main obstacle is the lack of enough knowledge in the theory of open threefolds. Topology works so far, but ultimately some geometric means become necessary.

This is a joint work with R.V. Gurjar, M. Koras, K. Masuda and P. Russell.

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