Mini courses
Workshop on the topology of algebraic varieties
Centre de recherches mathematiques
September, 2012

1. TOPOLOGICAL METHODS IN THE STUDY OF SINGULARITIES
(Anne Pichon and Walter Neumann)

The course will discuss the geometry and topology of isolated singularities of complex varieties, concentrating on dimensions 1 and 2, and working towards recent directions in bilipschitz geometry of singularities (last lecture). A tentative outline of the lectures is as follows:

1) Link of an isolated singularity, conicalness theorem, Milnor fibration. Complex plane curve singularities. Iterated torus links and Puiseux expansions.

2) Complex surface singularities, plumbing and resolution.

3) Graph manifolds. Graph manifold structure via resolution, Structure via Carrousel

4) Application to Lipschitz geometry of isolated singularities.

2. HARMONIC FUNCTIONS, KÄHLER MANIFOLDS, AND FUNDAMENTAL GROUPS
(Terrence Napier and Mohan Ramachandran)

Two fundamental facts in the theory of complex variables in the plane are that the real and imaginary parts of any holomorphic function are harmonic, and that any harmonic function on a simply connected region is the real part of a holomorphic function. This suggests that on a more general space, such as a Kähler manifold (a Kähler manifold is a complex manifold together with a Hermitian metric that behaves well with respect to the holomorphic structure), the (non)existence of harmonic functions with certain properties should provide information about the holomorphic, and even topological, structure of the space.

In this mini-course we will follow this line of thought mostly in the context of noncompact complete Kähler manifolds. As we will see, this approach, when applied to noncompact covering spaces, also leads to results regarding compact Kähler manifolds (such as smooth projective varieties) and in particular, their fundamental groups.

I. Potential theory on Riemannian manifolds.
1. Riemannian manifolds and (sub)harmonic functions.
2. Green's functions, hyperbolicity, and parabolicity.
3. The theorems of Sario and Nakai.

II. Potential theory on noncompact Kähler manifolds.
1. Complex manifolds and Kähler manifolds.
2. Holomorphic convexity, Stein manifolds, and pluri(sub)harmonic functions.
3. Bounded geometry.

III. Structure theorems for noncompact complete Kähler manifolds.
1. Ends.
2. The Bochner-Hartogs dichotomy.
3. Other results.

IV. Compact Kähler manifolds and their fundamental groups.

3. LOGARITHMIC GENERA AND THE BMY-INEQUALITY
(Adrian Langer and Ryoichi Kobayashi)

a) Langer's Lectures. The main aim of the lectures is to show a generalization of the Bogomolov-Miyaoka-Yau inequality to log canonical surfaces and applications of this inequality. We will start with definition of log canonical pairs and logarithmic forms. Then we use generalization of the logarithmic ramification formula to prove the Bogomolov-Sommese vanishing theorem. This plays an important role in proving the Bogomolov-Miyaoka-Yau inequality. We also plan to show how to compute local orbifold Euler numbers for quotient singularities. Finally, we plan to give some applications of the BMY inequality to problem of bounding curves on surfaces of general type.

Apart from classical references the following will be useful:

D. Greb, S. Kebekus, S. Kov?cs, T. Peternell, Differential forms on log canonical spaces, Publ. Math. Inst. Hautes ?tudes Sci. No. 114 (2011), 87?169.
A. Langer, Chern classes of reflexive sheaves on normal surfaces, Math. Z. 235 (2000), 591?614.
A. Langer, The Bogomolov-Miyaoka-Yau inequality for log canonical surfaces, J. London Math. Soc. 64 (2001), 327?343.
A. Langer, Logarithmic orbifold Euler numbers of surfaces with applications, Proc. London Math. Soc. 86 (2003), 358?396.
G. Megyesi, Generalisation of the Bogomolov-Miyaoka-Yau inequality to singular surfaces, Proc. London Math. Soc. 78 (1999), 241?282.
Y. Miyaoka, The orbibundle Miyaoka-Yau-Sakai inequality and an effective Bogomolov-McQuillan theorem, Publ. Res. Inst. Math. Sci. 44 (2008), 403?417.

b) Kobayashi's lectures. What I have in mind as a topic which is directly connected to the BMY inequality is the second main theorem (still a conjectire) in Nevanlinna theory. Here is an abstract.

The idea behind the conjectural second main theorem in Nevanlinna theory belongs to the same circle of ideas as BMY inequality. In my lecture, I will start with basic definitions and standard facts in this topic for beginners. Then I will introduce you the original R. Nevanlinna's second main theorem with a proof following R. Nevanlinna's original idea. The key is an algebro-geometric Lemma on logarithmic derivative. This clarifies the role of Gauss-Bonnet theorem (or the canonical bundle) in Nevanlinna's second main theorem. Then I will state and give a proof of Cartan's second main theorem for the approximation to hyperplanes of holomorphic curves in $P^n(C)$. The proof is again based on the algebro-geometric Lemma on logarithmic derivative. If I have time, I would like to say something about holomorphic curves in Abelian varieties.