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Talk Montreal: (17/10/2012)

$\underline{c}$-gluing construction & slices of QF space

MOTIVATIONS

[Thanks organizers, apology for people who have a different variation of this talk BUT I just discovered higher rank last year ... ↓

- Constructions I will describe good for finding a new parameterization for slices of QF or ∂QF
- → Pleating coord. Theory.
- → Plumbing constructions & non-variational coord. for moduli spaces

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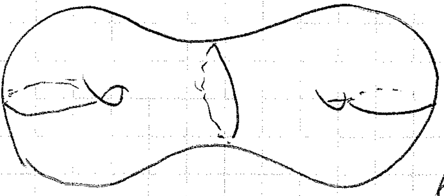


Σ surface s.t. $\chi(\Sigma) = \overset{=-k}{-(2g-2+b)} < 0$

$\xi(\Sigma) = 3g-3+b = \#$ pants curves

$\mathcal{P} = \{\sigma_1, \dots, \sigma_\xi\}$ $\{P_1, \dots, P_k\} = \mathcal{PC}$

A complex proj. str. on Σ is a (G,X)-str. on Σ w/

$G = PSl_2\mathbb{C}$ $X = \hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$

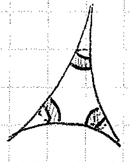
$\rightsquigarrow$ Dev: $\tilde{\Sigma} \longrightarrow \hat{\mathbb{C}}$

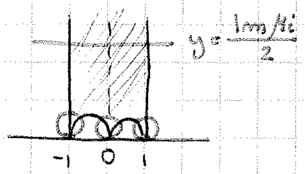
$\rho : \pi_1\Sigma \longrightarrow PSl_2\mathbb{C}$

GLUING CONSTRUCTION $\mu \in \mathbb{H}^\xi$

STEP 1 Every p.o.p. P is homeo to $\mathbb{H}^2/\Gamma$ where $\Gamma = \langle A, B\rangle$

$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$





We will identify each P_i w/ Δ:

$\phi_j : P_j \longrightarrow \mathbb{H}^2/\Gamma$

$\rightsquigarrow$ This labels the 2 hexagons of P_i (as black/white ... back/front) & the ∂-comp. (as $\partial_0 P_i$, $\partial_1 P_i$, $\partial_\infty P_i$)

$\underline{c}$ - GLUING CONSTRUCTION $\underline{c} \in (\mathbb{R}_+)^\xi$

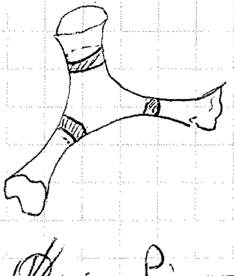
$\mu \in \left(\mathbb{C}/2\pi i\right)^\xi$

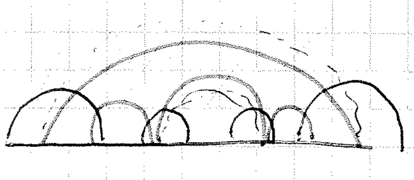
STEP 1 Every p.o.p P is homeo to $\mathbb{H}^2/\Gamma$ where $\Gamma = \langle \mathbb{A}, \mathbb{B} \rangle$

$$\mathbb{A} = \begin{pmatrix} ch\, c_1 & ch\, c_1 + 1 \\ ch\, c_1 - 1 & ch\, c_1 \end{pmatrix}$$

$$\mathbb{B} = \begin{pmatrix} ch\, c_2 & \coth\frac{c_1}{2}\, th\frac{\vartheta_1}{2}\, sh\, c_2 \\ th\frac{c_1}{2}\coth\frac{\vartheta_1}{2}\, sh\, c_2 & ch\, c_2 \end{pmatrix}$$

$$\underset{\underset{\cosh\delta_1}{\shortparallel}}{\coth\vartheta_1} = \frac{ch\, c_1\, ch\, c_2 - ch\, c_3}{sh\, c_1\, sh\, c_2}$$





$\phi_j : P_j \longrightarrow \mathbb{H}^2/\Gamma$

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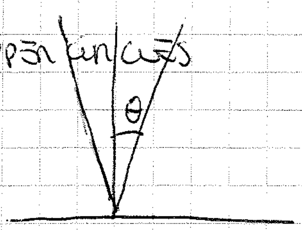
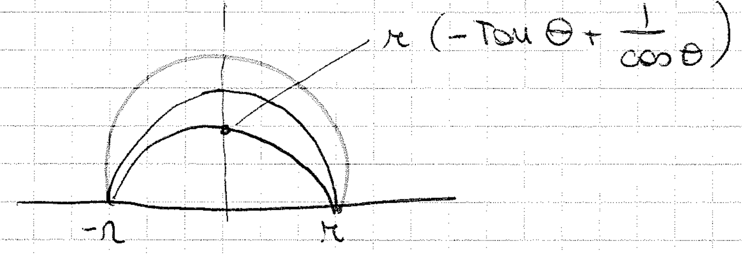
HYPERCIRCLES

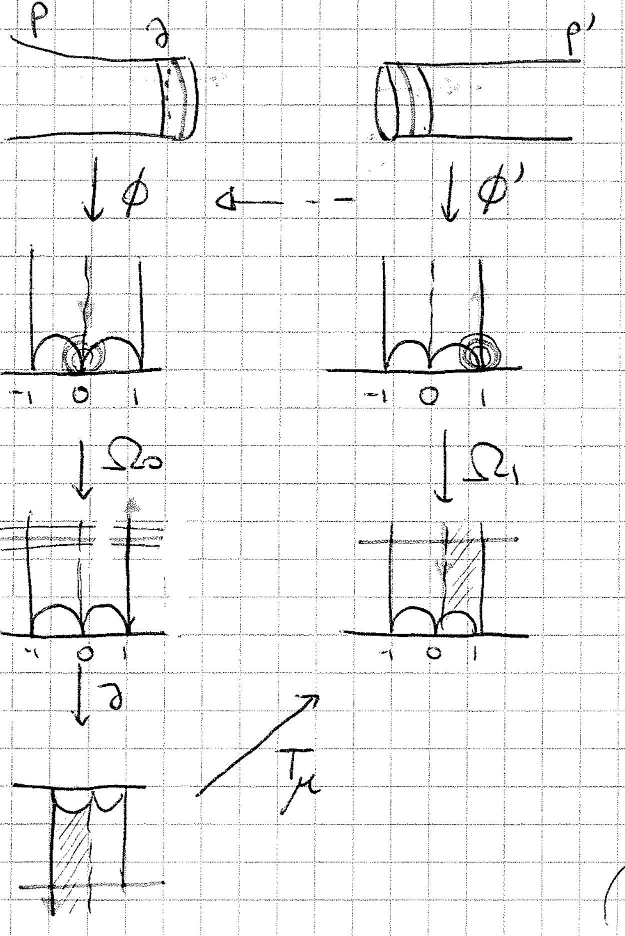




STEP 2 If $\sigma_i = \partial_\varepsilon P \cap \partial_{\varepsilon'} P'$, then the gluing is described by $\Omega_\varepsilon^{-1} J^{-1} T_{\mu_i}^{-1} \Omega_{\varepsilon'}$ where

$\Omega_0 = \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$ $\Omega_1 = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$

$J = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$ $T_{\mu_i} = i\begin{pmatrix} 1 & \mu_i \\ 0 & 1 \end{pmatrix}$

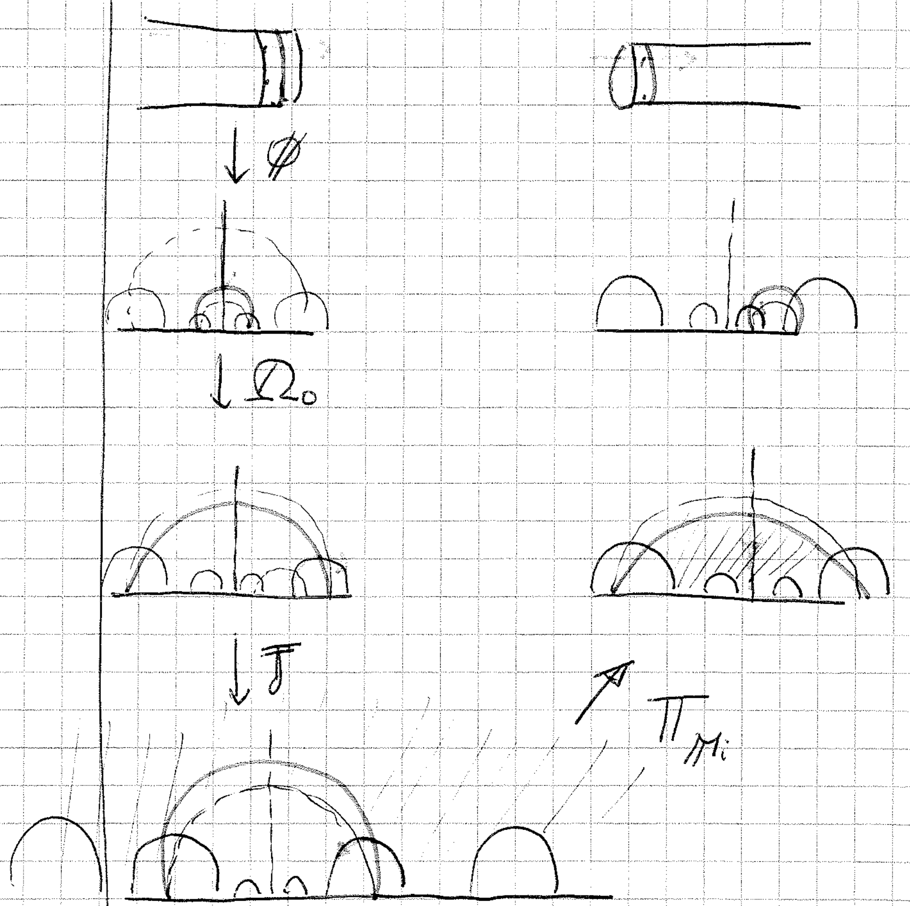


THM If Dev_μ is embedding, then ρ_μ is in the **MASKIT SLICE**

STEP 2 $\Omega_\varepsilon^{-1} \mathbb{J}^{-1} \Pi_{\mu_i}^{-1} \Omega_{\varepsilon'}$

$\mathbb{J} = \begin{pmatrix} 0 & -\mathrm{coth}\frac{c_i}{2} \\ \mathrm{th}\frac{c_i}{2} & 0 \end{pmatrix}$

$\Pi_{\mu_i} = \begin{pmatrix} \mathrm{ch}\frac{\mu}{2} & -\mathrm{sh}\frac{\mu}{2}\,\mathrm{coth}\frac{c_i}{2} \\ -\mathrm{sh}\frac{\mu}{2}\,\mathrm{th}\frac{c_i}{2} & \mathrm{ch}\frac{\mu}{2} \end{pmatrix}$



Q ?? → see next section!

RK Top term formula & Asymptotic direction

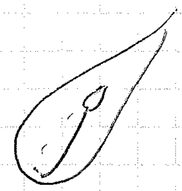
<u>THM</u> If $\underline{c}\to\underline{0}$ keeping $\underline{\mu}$ fixed when $\mu=\frac{-\boldsymbol{\mu}+i\pi}{c}$ then the $\underline{c}$-gluing construction (w/ param $\boldsymbol{\mu}\in\left(\mathbb{C}_+/2\pi i\right)^{\xi}$) converges to the pl.-construction (w/ " $\mu\in\mathbb{H}^{\xi}$)

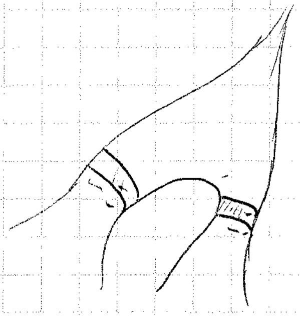
<u>RK</u> Idea come from a calculation made by Parker-Parkkonen in the case $\Sigma=\Sigma_{1,1}$ (generalisation of plumbing construction in Kra's style)

② For taking limit it is import the actual representative in the conj. class

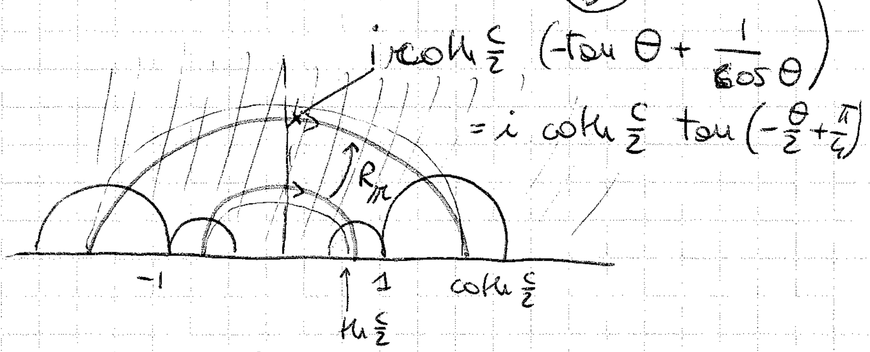
<u>IDEA OF PF</u> $\Sigma=\Sigma_{1,1}$ $\boldsymbol{\mu}\in\mathbb{C}/2\pi i$

$$\Gamma=\left\langle A=\begin{pmatrix}\mathrm{ch}\,c & \mathrm{ch}\,c+1\\ \mathrm{ch}\,c-1 & \mathrm{ch}\,c\end{pmatrix},\ B=\begin{pmatrix}\mathrm{ch}\,c & \mathrm{ch}\,c-1\\ \mathrm{ch}\,c+1 & \mathrm{ch}\,c\end{pmatrix}\right\rangle$$





$R_{\boldsymbol{\mu}}=\Big($



$$R_{\boldsymbol{\mu}}=\begin{pmatrix}\mathrm{ch}\frac{\boldsymbol{\mu}}{2}\coth\frac{c}{2} & -\mathrm{sh}\frac{\boldsymbol{\mu}}{2}\\ -\mathrm{sh}\frac{\boldsymbol{\mu}}{2} & \cosh\frac{\boldsymbol{\mu}}{2}\,\mathrm{th}\frac{c}{2}\end{pmatrix}$$

$$\begin{cases}\mathrm{Re}\,\mu=\frac{-\mathrm{Re}\,\boldsymbol{\mu}}{c}\\ \mathrm{Im}\,\mu=\frac{-\mathrm{Im}\,\boldsymbol{\mu}+\pi}{c}\end{cases}$$

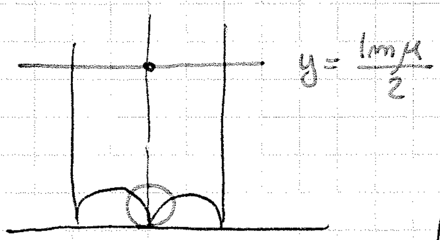
$G=\langle\Gamma,R_{\boldsymbol{\mu}}\rangle$ $\quad\mu=\frac{-\boldsymbol{\mu}+i\pi}{c}\leftarrow$ $\quad\boldsymbol{\mu}=i\pi-c\mu$

$$R_{\boldsymbol{\mu}}=\begin{pmatrix}-i\,\mathrm{sh}\frac{\mu c}{2}\coth\frac{c}{2} & -i\cosh\left(\frac{\mu c}{2}\right)\\ -i\cosh\left(\frac{\mu c}{2}\right) & -i\,\mathrm{sh}\left(\frac{\mu c}{2}\right)\mathrm{th}\frac{c}{2}\end{pmatrix}$$

$$A\to\begin{pmatrix}1&2\\0&1\end{pmatrix}$$

$$B\to\begin{pmatrix}1&0\\2&1\end{pmatrix}$$

$$R_{\boldsymbol{\mu}}\to-i\begin{pmatrix}\mu&1\\1&0\end{pmatrix}$$



$$\boxed{\begin{aligned}\mathrm{ch}\left(i\tfrac{\pi}{2}-x\right)&=-i\,\mathrm{sh}(x)\\ \mathrm{sh}\left(\quad\right)&=i\,\mathrm{ch}\,x\end{aligned}}$$

Ⅱ LINEAR SLICES

FN COOR → CLASSICAL $\quad FN_{\mathbb{R}} : \mathcal{T}(\Sigma) \longrightarrow (\mathbb{R}_+ \times \mathbb{R})^{\xi}$

→ COMPLEX $\quad FN_{\mathbb{C}} : Q\mathcal{T}(\Sigma) \longrightarrow \left(\mathbb{C}_{+}/_{2\pi i} \times \mathbb{C}/_{2\pi i}\right)^{\xi}$

$\boxed{\text{CASE } \Sigma = \Sigma_{1,1}}$ (KOMORI - YAMASHITA // KOMORI - PARKKONEN) $\quad c \in \mathbb{R}_+$

DEF $\mathcal{L}_c(\Sigma_{1,1}) = \{ \tau \in \mathbb{C}/_{2\pi i} \,/\, (c, \tau) \in FN_{\mathbb{C}}(QF) \}$

NOTE $\mathbb{R} \subset \mathcal{L}_c(\Sigma_{1,1})$

DEF $BM_c(\Sigma_{1,1})$ = c.c. of $\mathcal{L}_c(\Sigma_{1,1})$ containing Fuchsian locus

NOTE We have periodicity given by change of marking:

- $\tau \in \mathcal{L}_c \Rightarrow -\tau \in \mathcal{L}_c \qquad (D \leftrightarrow -D)$
- $\tau \in \mathcal{L}_c \Rightarrow \tau + 2ci \in \mathcal{L}_c \qquad (D \leftrightarrow T_6(D) = 6D)$

PROP (KEEN-SERIES) $BM_c = X_c^+ \underset{\exists}{\sqcup} X_c^-$ where

$X_c^{\pm} = \{ \tau \in \mathcal{L}_c \,/\, |pl^{\pm}(G_{c,\tau})| = 6 \}$

⤳ PLEATING COORD. THEORY!

PROP (KOMORI-YAMASHITA) • $\exists\, c_0, c_1 \in \mathbb{R}$ s.t. $\forall c \in (0, c_0)$ $\mathcal{L}_c$ is connected & $\forall c > c_1$ $\mathcal{L}_c$ is not connected

• $\mathcal{M} = \bigcup_{c>0} \mathcal{L}_c(\Sigma_{1,1})$

OPEN Q $\quad c_0 = c_1$?

Meaning of other c.c.?

$\boxed{\text{GENERAL CASE}}$

DEF 1 Fixing $\underline{c} \in \mathbb{R}_+^{\xi}$, we define

$\mathcal{L}_{\underline{c}}(\Sigma) = \{ \underline{\tau} \in (\mathbb{C}/_{2\pi i})^{\xi} \,/\, (\underline{c}, \underline{\tau}) \in FN_{\mathbb{C}}(QF),\ \operatorname{sgn}(\operatorname{Im}\tau_1) = \cdots = \operatorname{sgn}(\operatorname{Im}\tau$

DEF 2 Fixing $c \in \mathbb{R}_+$, we define

$\mathcal{L}_c(\Sigma) = \{ (\underline{c}, \underline{\tau}) \in \quad \times (\mathbb{C}/_{2\pi i})^{\xi} \,/\, (\underline{c}, \underline{\tau}) \in FN_{\mathbb{C}}(QF),\ \operatorname{Im}\tau_1 = \cdots = \operatorname{Im}\tau_{\xi},\ \sum c_i = c \}$

RK * Periodicity
* c.c. containing the Fuchsian locus

↗

OPEN Q

* Meaning of other C.C.
* Connectedness
* Convergence // Are slices nested
* Characterization of $BM_c \setminus \mathcal{J}$