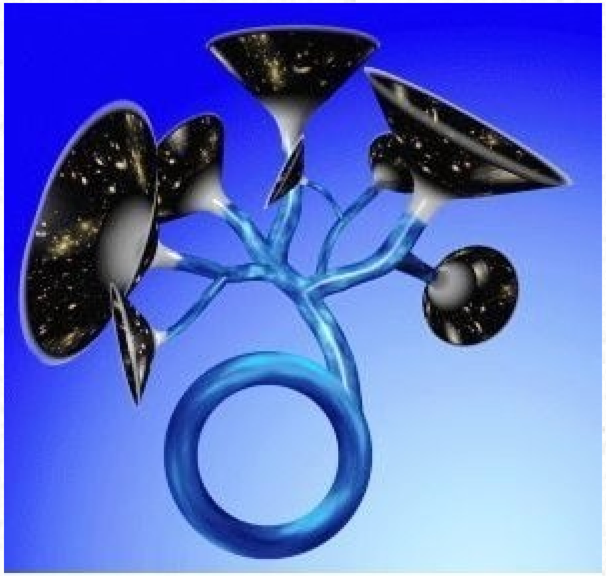
Particles in spacetimes in expansion



Richard Gott's self-creating universe

THÉORIE TEICHMÜLLER-THURSTON

D'ORDRE SUPÉRIEUR

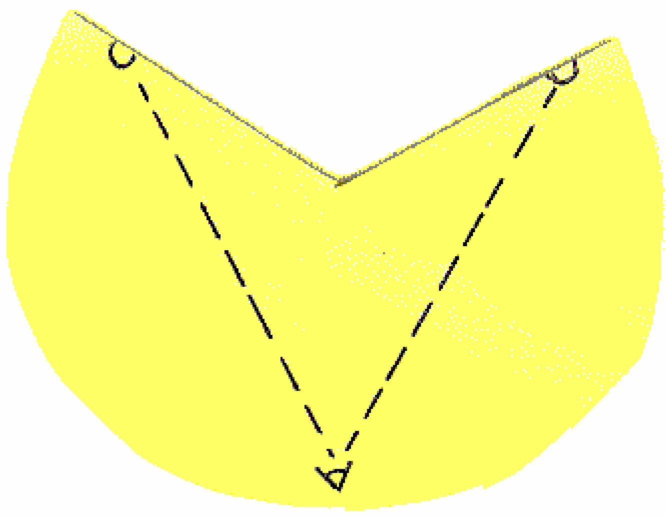
October 2012

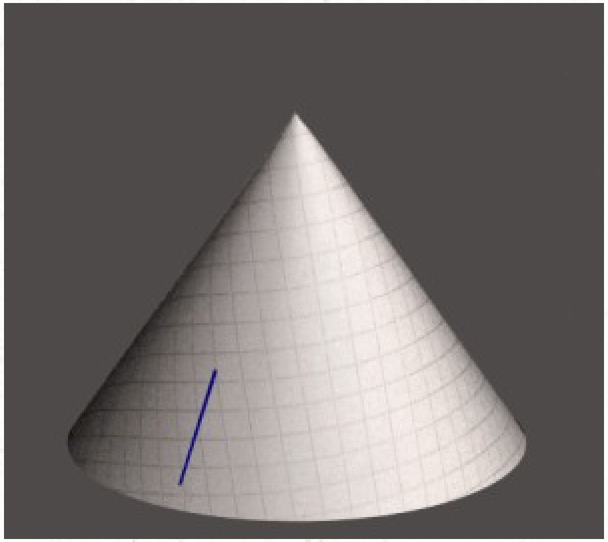
Thierry Barbot
Université d'Avignon

Joint work with C. Meusberger

Singularities in Riemannian manifolds

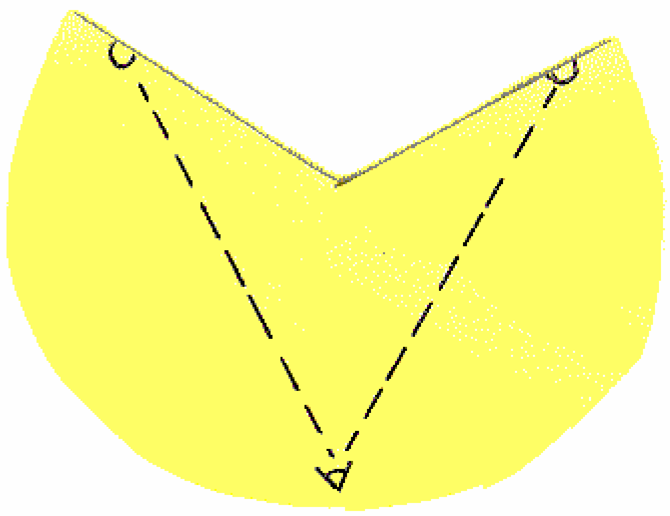
- **In dimension 2:** conical singularities

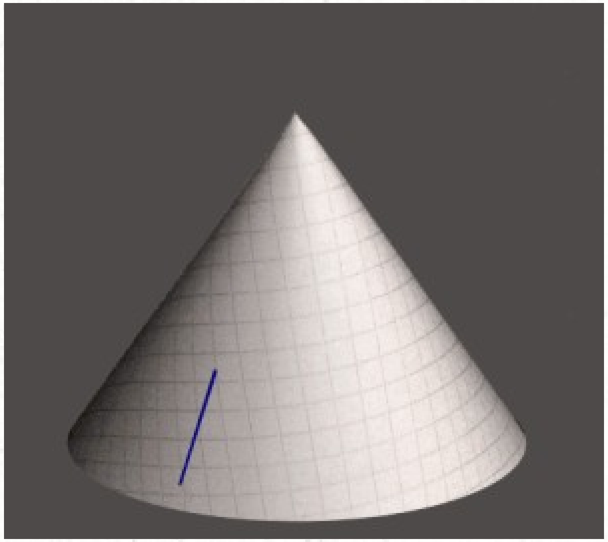




Singularities in Riemannian manifolds

- **In dimension 2:** conical singularities





- **In dimension 3:** (intersection of) singular lines

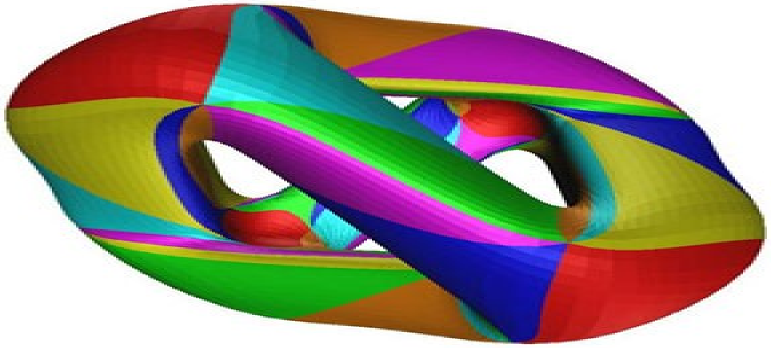
Theorem [Troyanov, 1991]:

Let S be a closed Riemann surface. Let $p_1, p_2, \ldots, p_n$ *be points in S and* $\theta_1, \theta_2, \ldots, \theta_n$ *positive real numbers such that:*

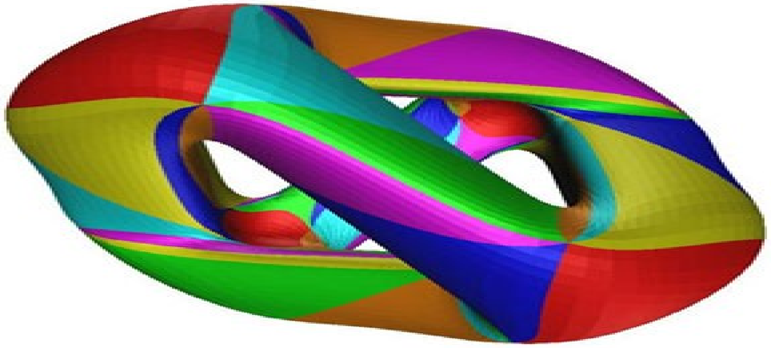
$$2\pi\chi(S) + (\theta_1 - 2\pi) + (\theta_2 - 2\pi) + \ldots + (\theta_n - 2\pi) < 0$$

Then, any negative-valued function on S is the curvature function of a unique metric in the given conformal class, such that each p_i *is a conical singularity of conical angle* θ_i*.*

Geometric construction through labeled fat graphs (or “decorated” triangulations)

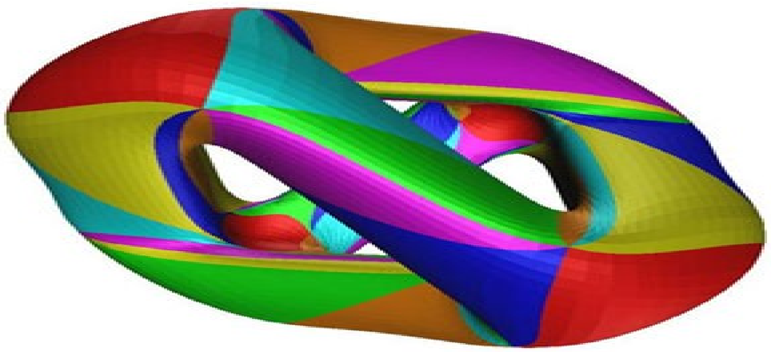


Geometric construction through labeled fat graphs (or "decorated" triangulations)



The data of positive real numbers at edges satisfying the triangular inequalities on each triangle encode singular hyperbolic (or euclidean) metrics on the surface: realize this data by hyperbolic (or euclidean) triangles and glue them.

Geometric construction through labeled fat graphs (or "decorated" triangulations)



The data of positive real numbers at edges satisfying the triangular inequalities on each triangle encode singular hyperbolic (or euclidean) metrics on the surface: realize this data by hyperbolic (or euclidean) triangles and glue them.

Dually, on can use the (free) data of angles at vertices. Cone angles can be prescribed - or not!

Fock-Chekhov Quantization

The length of the vertices are observables on the moduli space of singular metrics $\Rightarrow$ Poisson bracket

Fock-Chekhov Quantization

The length of the vertices are observables on the moduli space of singular metrics $\Rightarrow$ Poisson bracket

$\Rightarrow$ Quantum algebra (Fock-Chekhov, Bonahon & al.)

Fock-Chekhov Quantization

The length of the vertices are observables on the moduli space of singular metrics $\Rightarrow$ Poisson bracket

$\Rightarrow$ Quantum algebra (Fock-Chekhov, Bonahon & al.)

(actually, for the case where the singularities are cusps, but easily extended to the case of conical singularities, or hyperbolic ends)

Moduli space for the moduli space of flat $PSL(2, \mathbb{R})$ - connections (following Penner, Kashaev)

Σ **oriented** surface with "boundary" $c_1, \dots, c_n$.

Moduli space for the moduli space of flat PSL(2, $\mathbb{R}$) - connections (following Penner, Kashaev)

Σ **oriented** surface with "boundary" $c_1, \dots, c_n$.

$\Gamma = \pi_1(\Sigma)$; $\gamma_1, \dots, \gamma_n$ conjugacy classes in $G=PSL(2, \mathbb{R})$

Moduli space for the moduli space of flat $PSL(2, \mathbb{R})$ - connections (following Penner, Kashaev)

Σ **oriented** surface with "boundary" $c_1, \ldots, c_n$.

$\Gamma = \pi_1(\Sigma)$; $\gamma_1, \ldots, \gamma_n$ conjugacy classes in $G=PSL(2, \mathbb{R})$

$$\mathcal{M}_{\gamma_1, \ldots, \gamma_n} = \{\rho \in \mathrm{Rep}(\Sigma, G) \,/\, \rho(c_i) \in \gamma_i\}_{/G}$$

Moduli space for the moduli space of flat $PSL(2, \mathbb{R})$ - connections (following Penner, Kashaev)

Σ **oriented** surface with "boundary" $c_1, \ldots, c_n$.

$\Gamma = \pi_1(\Sigma)$; $\gamma_1, \ldots, \gamma_n$ conjugacy classes in $G=PSL(2, \mathbb{R})$

$$\mathcal{M}_{\gamma_1, \ldots, \gamma_n} = \{\rho \in \mathrm{Rep}(\Gamma, G) \,/\, \rho(c_i) \in \gamma_i\}_{/G}$$

$$= \{\text{Flat } G\text{-bundle over } \Sigma \ldots\}_{/\text{Gauge transf.}}$$

Moduli space for the moduli space of flat PSL(2, $\mathbb{R}$) - connections (following Penner, Kashaev)

Σ **oriented** surface with "boundary" $c_1, \dots, c_n$.

$\Gamma = \pi_1(\Sigma)$; $\gamma_1, \dots, \gamma_n$ conjugacy classes in G=PSL(2, $\mathbb{R}$)

$$\mathcal{M}_{\gamma_1, \dots, \gamma_n} = \{\rho \in \mathrm{Rep}(\Gamma, G) \,/\, \rho(c_i) \in \gamma_i\}_{/G}$$

$$= \{\text{Flat G-bundle over } \Sigma \ldots\}_{/\text{Gauge transf.}}$$

ρ-**admissible** triangulation T: if c_i, c_j are connected by an edge of T, then the subgroup of G generated by $\rho(c_i)$ and $\rho(c_j)$ is not solvable (no common fixed point in $\mathbb{H}^2$).

Theorem [Kashaev, 2003]:

For any element ρ *in* $\mathcal{M}_{\gamma_1, \ldots, \gamma_n}$*, there is a* ρ*-admissible triangulation* T.

Theorem [Kashaev, 2003]:
For any element ρ *in* $\mathcal{M}_{\gamma_1, \dots, \gamma_n}$, *there is a* ρ*-admissible triangulation* T.

Given a flat G-connection ρ over Σ. Let us consider the case where the γ_i's are elliptic. Let T be a ρ-admissible triangulation.

Theorem [Kashaev, 2003]:

For any element ρ *in* $\mathcal{M}_{\gamma_1, \ldots, \gamma_n}$, *there is a* ρ*-admissible triangulation* T.

Given a flat G-connection ρ over Σ. Let us consider the case where the γ_i's are elliptic. Let T be a ρ-admissible triangulation.

Trivialize the G-bundle over triangles $\Rightarrow g_{ij}$

Theorem [Kashaev, 2003]:

For any element ρ *in* $\mathcal{M}_{\gamma_1, \dots, \gamma_n}$, *there is a* ρ*-admissible triangulation* T.

Given a flat G-connection ρ over Σ. Let us consider the case where the γ_i's are elliptic. Let T be a ρ-admissible triangulation.

Trivialize the G-bundle over triangles $\Rightarrow g_{ij}$

For every triangle t, compute $\rho(a_i)$, where a_i is a loop starting in t and turning around c_i

Theorem [Kashaev, 2003]:
For any element ρ in $\mathcal{M}_{\gamma_1, \dots, \gamma_n}$, there is a ρ-admissible triangulation T.

Given a flat G-connection ρ over Σ. Let us consider the case where the γ_i's are elliptic. Let T be a ρ-admissible triangulation.

Trivialize the G-bundle over triangles $\Rightarrow g_{ij}$
For every triangle t, compute $\rho(a_i)$, where a_i is a loop starting in t and turning around c_i $\Rightarrow$ fixed point z_i of $\rho(a_i)$.

Theorem [Kashaev, 2003]:

For any element ρ *in* $\mathcal{M}_{\gamma_1, \dots, \gamma_n}$, *there is a* ρ*-admissible triangulation* T.

Given a flat G-connection ρ over Σ. Let us consider the case where the γ_i's are elliptic. Let T be a ρ-admissible triangulation.

Trivialize the G-bundle over triangles $\Rightarrow g_{ij}$

For every triangle t, compute $\rho(a_i)$, where a_i is a loop starting in t and turning around $c_i \Rightarrow$ fixed point z_i of $\rho(a_i)$.

$\Rightarrow$ hyperbolic triangle t_ρ

Theorem [Kashaev, 2003]:

For any element ρ *in* $\mathcal{M}_{\gamma_1, \dots, \gamma_n}$ *, there is a* ρ*-admissible triangulation* T.

Given a flat G-connection ρ over Σ. Let us consider the case where the γ_i's are elliptic. Let T be a ρ-admissible triangulation.

Trivialize the G-bundle over triangles $\Rightarrow$ g_{ij}

For every triangle t, compute $\rho(a_i)$, where a_i is a loop starting in t and turning around c_i $\Rightarrow$ fixed point z_i of $\rho(a_i)$.

$\Rightarrow$ hyperbolic triangle t_ρ

These hyperbolic triangles are non-degenerate (because T is ρ-admissible), and can be glued through the g_{ij}'s.

Theorem [Kashaev, 2003]:
For any element ρ *in* $\mathcal{M}_{\gamma_1, \ldots, \gamma_n}$, *there is a* ρ*-admissible triangulation* T.

Given a flat G-connection ρ over Σ. Let us consider the case where the γ_i's are elliptic. Let T be a ρ-admissible triangulation.

Trivialize the G-bundle over triangles $\Rightarrow g_{ij}$
For every triangle t, compute $\rho(a_i)$, where a_i is a loop starting in t and turning around c_i $\Rightarrow$ fixed point z_i of $\rho(a_i)$.
$\Rightarrow$ hyperbolic triangle t_ρ

These hyperbolic triangles are non-degenerate (because T is ρ-admissible), and can be glued through the g_{ij}'s.

Foldings?

There is no folding if for every t in T the orientation on $t \approx t_\rho$ induced by the orientation of $\mathbb{H}^2$ always coincide with the orientation induced by Σ (or its opposite).

There is no folding if for every t in T the orientation on $t \approx t_\rho$ induced by the orientation of $\mathbb{H}^2$ always coincide with the orientation induced by Σ (or its opposite).

$\Rightarrow$ (Kashaev) parametrization of $\mathcal{M}_{\gamma_1, \dots, \gamma_n}$ by $\mathbb{R}^a$ x $\{\pm 1\}^b$
(a= number of edges; b= number of triangles)

There is no folding if for every t in T the orientation on $t \approx t_\rho$ induced by the orientation of $\mathbb{H}^2$ always coincide with the orientation induced by Σ (or its opposite).

$\Longrightarrow$ (Kashaev) parametrization of $\mathcal{M}_{\gamma_1, \dots, \gamma_n}$ by $\mathbb{R}^a$ x $\{\pm 1\}^b$
(a= number of edges; b= number of triangles)

$\Longrightarrow$ Quantization.

Singularities in spacetimes: examples

Oriented Minkowski space

One massive particle

One lightlike singularity (graviton)

Gott's pair spacetime
(+ distance between particles)
(J.R. Gott, Phys. Rev. Lett. **66**, 1991)

Spin of the particle: the component of translation along the line

Theorem [S.N. Carroll, E. Farhi, A.H. Guth, 1992]:
Consider an open (2+1)-dimensional universe composed of point particles, each with a future-directed momentum that is either timelike or lightlike. If the total momentum of this universe is timelike, no subgroup of these particles can have a spacelike momentum.

Theorem [S.N. Carroll, E. Farhi, A.H. Guth, 1992]:
Consider an open (2+1)-dimensional universe composed of point particles, each with a future-directed momentum that is either timelike or lightlike. If the total momentum of this universe is timelike, no subgroup of these particles can have a spacelike momentum.

Problem: *How to mathematically settle this statement? What is an ``open (2+1)-dimensional universe composed of point particles"?*

Theorem [S.N. Carroll, E. Farhi, A.H. Guth, 1992]:
Consider an open (2+1)-dimensional universe composed of point particles, each with a future-directed momentum that is either timelike or lightlike. If the total momentum of this universe is timelike, no subgroup of these particles can have a spacelike momentum.

Problem: *How to mathematically settle this statement? What is an ``open (2+1)-dimensional universe composed of point particles"?*

Flat spacetime homeomorphic to $\mathbb{R}^3$

Theorem [S.N. Carroll, E. Farhi, A.H. Guth, 1992]:
Consider an open (2+1)-dimensional universe composed of point particles, each with a future-directed momentum that is either timelike or lightlike. If the total momentum of this universe is timelike, no subgroup of these particles can have a spacelike momentum.

Problem: *How to mathematically settle this statement? What is an ``open (2+1)-dimensional universe composed of point particles"?*

Flat spacetime homeomorphic to $\mathbb{R}^3$

Evolution process with time?

Theorem [S.N. Carroll, E. Farhi, A.H. Guth, 1992]:
Consider an open (2+1)-dimensional universe composed of point particles, each with a future-directed momentum that is either timelike or lightlike. If the total momentum of this universe is timelike, no subgroup of these particles can have a spacelike momentum.

Problem: *How to mathematically settle this statement? What is an ``open (2+1)-dimensional universe composed of point particles"?*

Flat spacetime homeomorphic to $\mathbb{R}^3$

Evolution process with time? No (closed timelike curves)

Theorem [S.N. Carroll, E. Farhi, A.H. Guth, 1992]:
Consider an open (2+1)-dimensional universe composed of point particles, each with a future-directed momentum that is either timelike or lightlike. If the total momentum of this universe is timelike, no subgroup of these particles can have a spacelike momentum.

Problem: *How to mathematically settle this statement? What is an ``open (2+1)-dimensional universe composed of point particles"?*

Flat spacetime homeomorphic to $\mathbb{R}^3$

Evolution process with time? No (closed timelike curves)

Geodesic completeness?

Theorem [S.N. Carroll, E. Farhi, A.H. Guth, 1992]:
Consider an open (2+1)-dimensional universe composed of point particles, each with a future-directed momentum that is either timelike or lightlike. If the total momentum of this universe is timelike, no subgroup of these particles can have a spacelike momentum.

Problem: *How to mathematically settle this statement? What is an ``open (2+1)-dimensional universe composed of point particles"?*

Flat spacetime homeomorphic to $\mathbb{R}^3$

Evolution process with time? No (closed timelike curves)

Geodesic completeness? No (non-complete closed photons)

Theorem [S.N. Carroll, E. Farhi, A.H. Guth, 1992]:
Consider an open (2+1)-dimensional universe composed of point particles, each with a future-directed momentum that is either timelike or lightlike. If the total momentum of this universe is timelike, no subgroup of these particles can have a spacelike momentum.

Problem: *How to mathematically settle this statement? What is an ``open (2+1)-dimensional universe composed of point particles"?*

Flat spacetime homeomorphic to $\mathbb{R}^3$

Evolution process with time? No (closed timelike curves)

Geodesic completeness? No (non-complete closed photons)

Timelike geodesic completeness?

Theorem [S.N. Carroll, E. Farhi, A.H. Guth, 1992]:
Consider an open (2+1)-dimensional universe composed of point particles, each with a future-directed momentum that is either timelike or lightlike. If the total momentum of this universe is timelike, no subgroup of these particles can have a spacelike momentum.

Problem: *How to mathematically settle this statement? What is an ``open (2+1)-dimensional universe composed of point particles"?*

Flat spacetime homeomorphic to $\mathbb{R}^3$

Evolution process with time? No (closed timelike curves)

Geodesic completeness? No (non-complete closed photons)

Timelike geodesic completeness? Perharps

Theorem [S.N. Carroll, E. Farhi, A.H. Guth, 1992]:
Consider an open (2+1)-dimensional universe composed of point particles, each with a future-directed momentum that is either timelike or lightlike. If the total momentum of this universe is timelike, no subgroup of these particles can have a spacelike momentum.

Problem: *How to mathematically settle this statement? What is an ``open (2+1)-dimensional universe composed of point particles"?*

Flat spacetime homeomorphic to $\mathbb{R}^3$

Evolution process with time? No (closed timelike curves)

Geodesic completeness? No (non-complete closed photons)

Timelike geodesic completeness? Perharps

Data at infinity? Space of timelike geodesics?

Additional difficulties:

Other topology?

Other singularities:

Tachyon

Misner line (Big-Bang)

Misner line of second type (surrounded by CTC's)

Particle with spin

The anti-de Sitter Gott Universe

[Matschull, Holst; Class. Quantum Grav., **16**, 1999]

Anti-de Sitter space

Anti-de Sitter Gott Universe

Description of the domain filled by CTCs

A schematic view

More precisely: this is a region, avoiding the gravitons, admitting as frontier (the chronology horizon) the union of two null half-planes, both supported by the closed spacelike geodesic in the spacetime.

Creation of a BTZ black-hole

A BTZ black-hole

Obtained by removing some part of the region filled by CTCs, but containing no CTC.

Globally hyperbolic singular spacetimes

Let us avoid for the moment the causality pathologies.

Globally hyperbolic singular spacetimes

Let us avoid for the moment the causality pathologies.

Spinless, massive particles,

Locally flat (or locally AdS) spacetime which is *globally hyperbolic* and *spatially compact:* there is a time function with compact levels.

Let Σ denote a surface of genus $g \geq 2$ with n marked points $x_1, \ldots, x_n$.

Definition: *Given $\theta := (\theta_1, \ldots, \theta_n) \in (0, \pi)^n$, let $GH(\Sigma, \theta)$ be the space of globally hyperbolic singular AdS metrics on $\Sigma \times \mathbb{R}$, with cone singularities at the lines $\{x_i\} \times \mathbb{R}$ of angle θ_i for $1 \leq i \leq n$, considered up to isotopies fixing the singular lines.*

Let $Teich(\Sigma, \theta)$ be the Teichmüller space of hyperbolic metrics on Σ with conical singularity at x_i of angle θ_i for $1 \leq i \leq n$.

Let Σ denote a surface of genus $g \geq 2$ with n marked points $x_1, \ldots, x_n$.

Definition: *Given $\theta := (\theta_1, \ldots, \theta_n) \in (0, \pi)^n$, let $GH(\Sigma, \theta)$ be the space of globally hyperbolic singular AdS metrics on $\Sigma \times \mathbb{R}$, with cone singularities at the lines $\{x_i\} \times \mathbb{R}$ of angle θ_i for $1 \leq i \leq n$, considered up to isotopies fixing the singular lines.*

Let $Teich(\Sigma, \theta)$ be the Teichmüller space of hyperbolic metrics on Σ with conical singularity at x_i of angle θ_i for $1 \leq i \leq n$.

Theorem [Bonsante, Schlenker (2007)]: *There is a natural homeomorphism between $GH(\Sigma, \theta)$ and $Teich(\Sigma, \theta) \times Teich(\Sigma, \theta)$.*

The space of timelike geodesics of AdS is isomorphic to $\mathbb{H}^2 \times \mathbb{H}^2$. The homeomorphism above can be seen as the map associating to a (singular) spacetime M the (singular) space of timelike geodesics in M.

Let Σ denote a surface of genus $g \geq 2$ with n marked points $x_1, ..., x_n$.

Definition: *Given $\theta := (\theta_1, \dots, \theta_n) \in (0, \pi)^n$, let $GH(\Sigma,\theta)$ be the space of globally hyperbolic singular AdS metrics on $\Sigma \times \mathbb{R}$, with cone singularities at the lines $\{x_i\} \times \mathbb{R}$ of angle θ_i for $1 \leq i \leq n$, considered up to isotopies fixing the singular lines.*

Let $Teich(\Sigma,\theta)$ be the Teichmüller space of hyperbolic metrics on Σ with conical singularity at x_i of angle θ_i for $1 \leq i \leq n$.

Theorem [Bonsante, Schlenker (2007)]: *There is a natural homeomorphism between $GH(\Sigma,\theta)$ and $Teich(\Sigma,\theta) \times Teich(\Sigma,\theta)$.*

The space of timelike geodesics of AdS is isomorphic to $\mathbb{H}^2 \times \mathbb{H}^2$. The homeomorphism above can be seen as the map associating to a (singular) spacetime M the (singular) space of timelike geodesics in M.

There is also a natural identification between $GH(\Sigma,\theta)$ and $T^*Teich(\Sigma,\theta)$ through CMC foliations (Krasnov, Schlenker (2006)).

Particles with spin in globally hyperbolic flat spacetimes

Static spacetimes (spinless): product of a singular euclidean surface with $\mathbb{R}$ (the time).

Particles with spin in globally hyperbolic flat spacetimes

Static spacetimes (spinless): product of a singular euclidean surface with $\mathbb{R}$ (the time).

Stationary spacetimes: singular euclidian surface (Σ, g), and a closed 1-form ω outside on Σ^* (ie outside the singular points).

Particles with spin in globally hyperbolic flat spacetimes

Static spacetimes (spinless): product of a singular euclidean surface with $\mathbb{R}$ (the time).

Stationary spacetimes: singular euclidian surface (Σ, g), and a closed 1-form ω outside on Σ^* (ie outside the singular points).

$g = g - (\omega + dt)^2$ is a singular locally flat lorentzian metric on $\Sigma \times \mathbb{R}$.

Particles with spin in globally hyperbolic flat spacetimes

Static spacetimes (spinless): product of a singular euclidean surface with $\mathbb{R}$ (the time).

Stationary spacetimes: singular euclidian surface (Σ, g), and a closed 1-form ω outside on Σ^* (ie outside the singular points).

$g = g - (\omega + dt)^2$ is a singular locally flat lorentzian metric on $\Sigma \times \mathbb{R}$.

The singular lines $l_i = \{x_i\} \times \mathbb{R}$ are massive particles with spin, surrounded by closed timelike curves.

Particles with spin in globally hyperbolic flat spacetimes

Static spacetimes (spinless): product of a singular euclidean surface with $\mathbb{R}$ (the time).

Stationary spacetimes: singular euclidian surface (Σ, g), and a closed 1-form ω outside on Σ^* (ie outside the singular points).

$g = g - (\omega + dt)^2$ is a singular locally flat lorentzian metric on $\Sigma \times \mathbb{R}$.

The singular lines $l_i = \{x_i\} \times \mathbb{R}$ are massive particles with spin, surrounded by closed timelike curves.

Let $r_i = \frac{2\pi}{\theta_i} \int \omega$. Outside the cylinders of radius r_i around each l_i

- the **critical cylinders** - there is no closed timelike curves.

Theorem [B., Meusberger; 2011] *The complement in* $\Sigma \mathrm{x} \mathbb{R}$ *of the union of the critical cylinders is globally hyperbolic. Moreover, every* ***stationary*** *locally flat "globally hyperbolic" spacetime is obtained in this way.*

Theorem [B., Meusberger; 2011] *The complement in* $\Sigma \times \mathbb{R}$ *of the union of the critical cylinders is globally hyperbolic. Moreover, every* ***stationary*** *locally flat "globally hyperbolic" spacetime is obtained in this way.*

Our work also include a study of the measurement by teams of stationary observers of "time paradox" effects.

Theorem [B., Meusberger; 2011] *The complement in* $\Sigma \times \mathbb{R}$ *of the union of the critical cylinders is globally hyperbolic. Moreover, every* ***stationary*** *locally flat "globally hyperbolic" spacetime is obtained in this way.*

Our work also include a study of the measurement by teams of stationary observers of "time paradox" effects.

Non-stationary spacetimes with particles with spin?

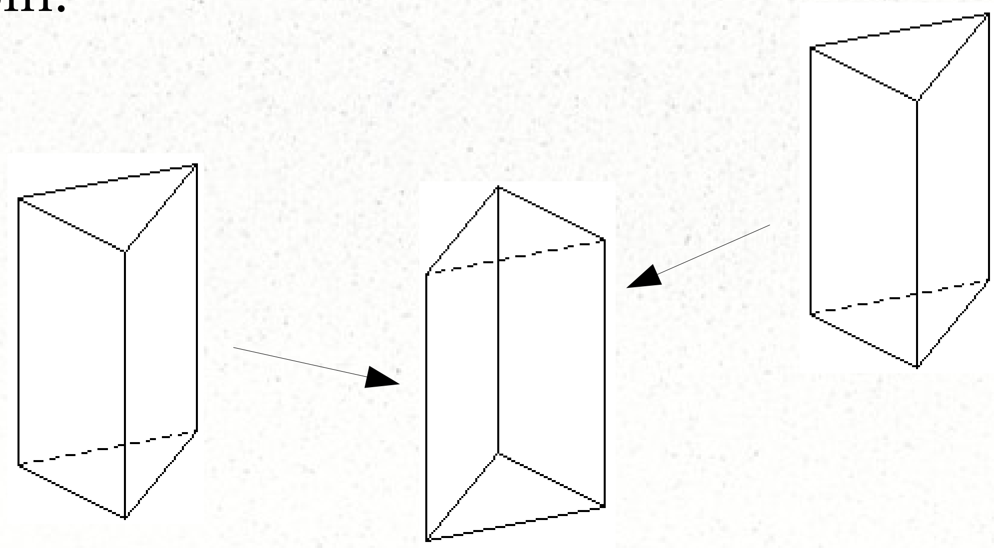
Another way to construct stationary spacetimes:

The euclidean surface (Σ, g) is obtained by glueing along their sides euclidean triangles.

Another way to construct stationary spacetimes:

The euclidean surface (Σ, g) is obtained by glueing along their sides euclidean triangles.

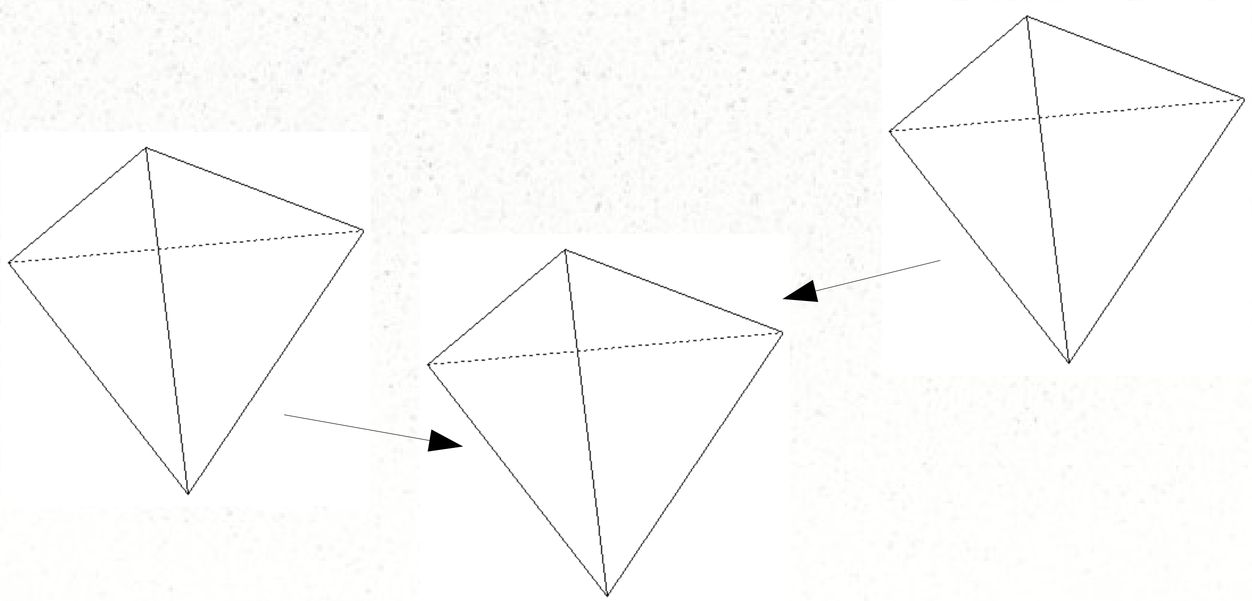
The product of these triangles by $\mathbb{R}$ are prisms that we can glue along their sides. We can translate in the vertical direction $\Longrightarrow$ spin.



Let now (Σ, g) be a singular hyperbolic surface. Then $g = t^2 g - dt^2$ is a locally flat singular lorentzian metric, where the singular lines are spinless massive particles. It is the **cone** over (Σ, g).

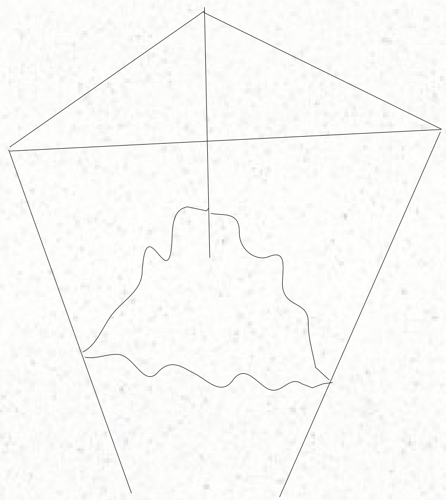
Let now (Σ, g) be a singular hyperbolic surface. Then $g = t^2 g - dt^2$ is a locally flat singular lorentzian metric, where the singular lines are spinless massive particles. It is the **cone** over (Σ, g).

Every hyperbolic triangle define a future complete tetraedron in the Minkowski space, and the cone is obtained by glueing these tetraedra.



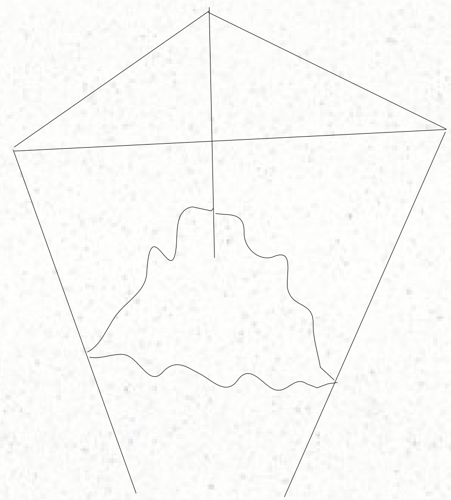
Truncated tilted tetraedra:

Given three non-parallel timelike lines in Minkowski:



Truncated tilted tetraedra:

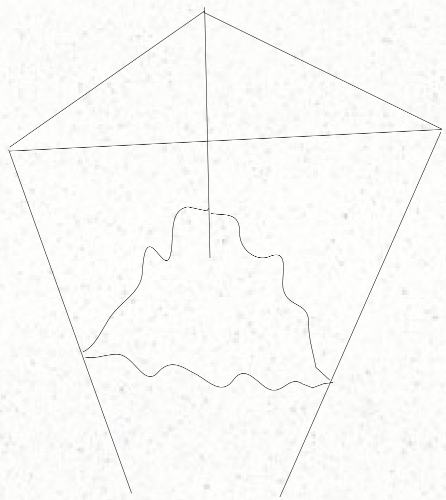
Given three non-parallel timelike lines in Minkowski:



Encoded by the hyperbolic triangle (one positive number on each edge) + a real number on each edge.

Truncated tilted tetraedra:

Given three non-parallel timelike lines in Minkowski:



Encoded by the hyperbolic triangle (one positive number on each edge) + a real number on each edge.

⇒ particles with spin in flat spacetimes, which are globally hyperbolic outside critical cylinders.

Parametrization “à la Kashaev” of the space of representations of $\pi_1(\Sigma)$ into the Poincaré group (affine isometries of the Minkowski space $\approx SO_0(1,2) \rhd \mathbb{R}^{1,2}$: data on each edge of an admissible triangulation of a pair (l, α) where l is a positive number (hyperbolic length of the edge) and α a number (positive or negative) the **signed distance** between the timelike lines.

Parametrization “à la Kashaev” of the space of representations of $\pi_1(\Sigma)$ into the Poincaré group (affine isometries of the Minkowski space $\approx SO_0(1,2) \ltimes \mathbb{R}^{1,2}$: data on each edge of an admissible triangulation of a pair (l, α) where l is a positive number (hyperbolic length of the edge) and α a number (positive or negative) the **signed distance** between the timelike lines.

$\Longrightarrow$ Quantization.

Parametrization "à la Kashaev" of the space of representations of $\pi_1(\Sigma)$ into the Poincaré group (affine isometries of the Minkowski space $\approx SO_0(1,2) \rtimes \mathbb{R}^{1,2}$: data on each edge of an admissible triangulation of a pair (l, α) where l is a positive number (hyperbolic length of the edge) and α a number (positive or negative) the **signed distance** between the timelike lines.

$\Longrightarrow$ Quantization.

THANKS FOR YOUR ATTENTION!