

ÉCOLE D'ÉTÉ SMS 2012 « COMBINATOIRE PROBABILISTE »  
25 JUIN - 6 JUILLET 2012

SMS 2012 SUMMER SCHOOL "PROBABILISTIC COMBINATORICS"  
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## Markov chains for graph colouring (3 hours)

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This series of talks will be a survey of results related to Markov Chain Monte Carlo algorithms for generating a random  $k$ -coloring of a graph with maximum degree  $D$ . We'll begin with the coupling technique, and its refinement known as path coupling due to Bubley-Dyer. Our starting point will be Jerrum's rapid mixing result of the simplest Markov chain (aka Glauber dynamics) when  $k > 2D$ . We'll then look at various improvements beginning with an improvement to  $k > (11/6)D$  using a more complicated chain that flips 2-color components. We'll see how to utilize properties of random colorings and use a multi-step coupling to get improved results assuming lower bounds on the girth and on  $D$ . And we'll see improved results for planar graphs by using the spectral radius. Finally, we'll look at recent improvements in algorithms for approximately counting colorings based on random sampling.

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