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Affine PBW bases and MV polytopes in rank 2

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MV (Mirkovic–Vilonen) polytopes appear in a variety of different guises. Anderson first defined them as the moment map images of MV cycles in the affine Grassmannian. Kamnitzer provided an explicit description of MV polytopes, and in giving this description, he discovered that MV polytopes encode precisely the reparameterization data of PBW bases. Later work of Baumann-Kamnitzer showed a connection with Lusztig's varieties of nilpotent preprojective algebra representations.

A natural question is to ask how much of this picture generalizes to the affine case. Beck-Chari-Pressley, Akasaka, and Beck-Nakajima define PBW bases for all affine types. Recent work of Baumann-Kamnitzer-Tingley give a geometric definition of affine MV polytopes in all untwisted affine types using Lusztig's nilpotent varieties. Also, work of Baumann–Dunlap–Kamnitzer–Tingley gives a combinatorial definition of MV polytopes for the two rank 2 affine cases. However, unlike in the finite-type case, it is not at all clear that these definitions coincide.

In this talk, I will discuss recent work (joint with P. Tingley) that addresses this question. We show that all three notions of MV polytopes coincide in two rank-2 affine cases. Perhaps even more interesting than the statement of the result is the method of proof. Explicitly checking the equations of affine MV polytopes is too unwieldy. Instead, we give a short list of axioms for affine MV polytopes and check these axioms in the three cases. In particular, our methods give a new proof of the analogous result in the finite-type case and shed some light on why MV polytopes are so prevalent.

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