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Applications of vertex operator constructions and character theory
to branching problems for level-1 representations of affine
Kac–Moody algebras : $D_4^{(1)}$ to $G_2^{(1)}$ and $E_6^{(1)}$ to $F_4^{(1)}$

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In 1991 Feingold, Frenkel and Ries gave a spinor construction of the vertex operator para-algebra $V = V^0 \oplus V^1 \oplus V^2 \oplus V^3$, whose summands are 4 level-1 irreducible representations (irreps) of the affine Kac-Moody (KM) algebra $D_4^{(1)}$. The triality group $S_3 = \langle \sigma, \tau \mid \sigma^3 = 1 = \tau^2, \tau\sigma\tau = \sigma^{-1} \rangle$ in $Aut(V)$ was constructed, preserving V^0 and permuting V^i , $i = 1, 2, 3$. V is $\frac{1}{2}\mathbb{Z}$ -graded and V_n^i denotes the n -graded subspace of V^i . Vertex operators $Y(v, z)$ for $v \in V_1^0$ represent $D_4^{(1)}$ on V , while those for which $\sigma(v) = v$ represent $G_2^{(1)}$. V decomposes into the direct sum of $G_2^{(1)}$ irreps by a two-step process, first decomposing with respect to the intermediate algebra $B_3^{(1)}$ represented by $Y(v, z)$ for $\tau(v) = v$. There are three vertex operators, $Y(\omega_{D_4}, z)$, $Y(\omega_{B_3}, z)$, $Y(\omega_{G_2}, z)$, each representing the Virasoro algebra given by the Sugawara constructions from the three algebras. These give two coset Virasoro constructions, $Y(\omega_{D_4} - \omega_{B_3}, z)$ and $Y(\omega_{B_3} - \omega_{G_2}, z)$, with central charges $1/2$ and $7/10$, respectively, the first commuting with $B_3^{(1)}$, the second commuting with $G_2^{(1)}$, and each commuting with the other. This gives the space of highest weight vectors for $G_2^{(1)}$ in V as sums of tensor products of irreducible Virasoro modules $L(1/2, h_1) \otimes L(7/10, h_2)$.

The dissertation research of my student, Quincy Loney, explicitly constructs these coset Virasoro operators, and uses them to study the decomposition of V with respect to $G_2^{(1)}$. The character theory of affine KM algebras and of the Virasoro algebra, is used to prove that the summands explicitly found fill V , providing the complete decomposition. This requires using some identities

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among the Rogers-Ramanujan (RR) series found by Ramanujan in his famous notebook.

The dissertation research of another student, Christopher Mauriello, carries out a similar project for the decomposition of the three level-1 irreps of $E_6^{(1)}$ with respect to its subalgebra $F_4^{(1)}$ of fixed points under the order 2 Dynkin diagram automorphism. That investigation uses the bosonic lattice construction, and gives the highest weight vectors for $F_4^{(1)}$ as sums of Virasoro modules $L(4/5, h)$ which form a W -algebra and modules for it. New character identities for the RR series are involved in showing that the summands found by explicit vertex operator computations provide the complete decomposition.