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WORKSHOP ON CONVEXITY AND ASYMPTOTIC GEOMETRIC ANALYSIS
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Bonnesen-style inradius inequalities

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Let $E \subset \mathbb{R}^n$ be a convex body with interior points and B_n the n-dimensional unit ball. The Bonnesen–Blaschke inequality for a planar convex body K establishes that

$$\mathrm{W}_1(K;E)^2 - \mathrm{V}(K)\mathrm{V}(E) \geq \frac{\mathrm{V}(E)^2}{4}(\mathrm{R}(K;E) - \mathrm{r}(K;E))^2 \tag{1}$$

where $\mathrm{W}_1(K;E)$ is the first quermassintegral of K w.r.t. E and $\mathrm{r}(K;E)$ and $\mathrm{R}(K;E)$ are the inradius and the circumradius of K w.r.t. E.

An extension of Bonnesen's inradius inequality to higher dimensions was conjectured by Wills and proved simultaneously by Bokowski and Diskant for $E = B_n$:

$$\mathrm{V}(K) - n\,\mathrm{r}(K;B_n)\mathrm{W}_1(K;B_n) + (n-1)\mathrm{r}(K;B_n)^n\mathrm{V}(B_n) \leq 0. \tag{2}$$

Sangwine-Yager proved it for a general relative body E with interior points, as a consequence of a much more general result which bounded the volume of every inner parallel body of K in terms of the quermassintegrals of K and some mixed volumes involving inner parallel bodies.

We provide new inequalities for the volume of (the inner parallel bodies of) a convex body in terms of the quermassintegrals of it, using the technique of inner parallel bodies. These bounds are obtained as consequences of, on the one hand, inequalities for inner parallel bodies involving mixed volumes and, on the other hand, inequalities which relate a convex body with its inner parallel bodies, its kernel and its form body.

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