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Volume bounds for shadow covering

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Suppose that K and L are compact convex subsets of n-dimensional Euclidean space, and suppose that, for every direction u, the orthogonal projection (that is, the shadow) of L onto the subspace $u^{\perp}$ contains a translate of the corresponding projection of K.

This covering condition does *not* imply that L contains a translate of K. In fact, it is even possible for L to have strictly smaller volume. This leads to several questions :

– When does shadow covering imply that L contains a translate of K ?
– What does shadow covering imply more generally about covering relations between K and L ?
– When does shadow covering imply that $V(K) \leq V(L)$?
– What does shadow covering imply more generally about the volume ratio $\frac{V(K)}{V(L)}$?

In this talk I will give a concrete construction for a large family convex sets K and L in n-dimensional Euclidean space such that each $(n-1)$-dimensional shadow of L contains a translate of the corresponding shadow of K, while at the same time K has strictly greater volume than L. This construction turns out to be sufficiently straightforward that a talented person could conceivably mold 3-dimensional examples out of modeling clay.

We will also show that, if the orthogonal projection L_u onto the subspace $u^{\perp}$ contains a translate of K_u for every direction u, then the set $\frac{n}{n-1}L$ contains a translate of K. It follows that

$$V_n(K) \leq \left(\frac{n}{n-1}\right)^n V_n(L).$$

In particular, we derive a universal constant bound,

$$V_n(K) \leq 2.942\, V_n(L),$$

independent of the dimension n of the ambient space. Related results are obtained for projections onto subspaces of some fixed intermediate co-dimension. Open questions and conjectures are also posed.

Some of these results arise from *joint work with Christina Chen and Tanya Khovanova*.

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