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Thematic Year 2012 – 2013
Moduli Spaces, Extremality and Global Invariants

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The thematic theme year will be dedicated to the study of the fundamental questions which lie at the heart of the current developments in geometry and topology from the point of view moduli spaces, extremality and global invariants. It will be held at the CRM from July 2012 to July 2013.

Geometry in the large sense, and especially the geometry of the most natural and simple structures on manifolds, is going through a golden age. More than ever, we are close to a satisfactory understanding of the basic blocks that constitute the most intricate spaces, be they Riemannian, complex, algebraic, symplectic or dynamical. Contemplating this development, one sees that moduli spaces play a ubiquitous role as a framework in which to analyze fundamental problems, while extremal methods and global invariants are often at the core of the solution of these problems. Moduli spaces arise in many contexts: as spaces of metrics, be they Kähler or almost Kähler; as spaces of almost complex structures; as spaces of classes of representations of the fundamental group of surfaces or 3-manifolds with values in suitable Lie groups, including the higher Teichmüller spaces; as the Hamiltonian diffeomorphism group over which the Hofer norm is given; as spaces of configurations of clusters, a sort of dual to the Fukaya A_∞-structure; as spaces of pseudo-algebraic functions on algebraic curves that are mysteriously connected to integrable systems, random matrices, and Gromov – Witten-invariants. In low dimensions, extremal methods and global invariants are at the heart of Perelman's proof of the geometrization conjecture, and in particular, the Poincaré conjecture. They are also present in higher dimensions, for instance in the classification problem for Kähler – Einstein metrics and constant scalar curvature Kähler metrics, where Aubin, Yau, Tian and Donaldson have established basic existence theorems. Global invariants such as those emanating from Floer theory are now percolating in dimensions 3 and 4. For instance, they are behind the recent proof by Taubes of the full Weinstein conjecture on the existence of closed orbits of the Reeb flow on 3-dimensional contact manifolds. In symplectic topology and geometry, the progress over the last decade is remarkable on almost all important questions: relative symplectic field theory is developing quickly and almost independently, the theory of J-holomorphic curves has been used to derive real enumerative invariants which has led to major advances in real algebraic geometry.

The thematic year's program consists of eight research workshops paired with mini-courses and four series of Aisenstadt lectures, all coordinated to maximize constructive interactions.

Workshops

The first workshop on *Spectral Invariants on Non-compact and Singular Spaces* will be held July 23 – 27, 2012, and is organized by Pierre Albin (Urbana-Champaign) and Frédéric Rochon (ANU & UQÀM). It will focus on the development and use of spectral invariants on non-compact and singular spaces, a very active area of contemporary research with many promising avenues. The workshop will stimulate the exchange of ideas between experts dealing with spectral invariants in different, but related, geometric contexts with the hope this will lead to new discoveries.

The second workshop, on *The Topology of Algebraic Varieties*, will be held on September 21 – 28, 2012. It is organized by Rajendra Gurjar (TIFR Mumbai), Shulim Kaliman (Miami), Steven Lu (UQÀM) and Peter Russell (McGill) and focuses on the connections between topological invariants of possibly non-complete normal complex varieties and other invariants, especially those coming from birational geometry and affine algebraic geometry. Of particular interest are: fundamental groups in connection with the Shafarevich problem; the interaction between algebraic and topological properties of open varieties; the study of the topology of surface singularities with its connections to 3-manifolds; the relation of the Bogomolov – Miyaoka – Yau inequality, the (logarithmic) Chern numbers of these surfaces and local data coming from the singular points. To complete the program, a series of Aisenstadt Chair lectures will be delivered by Fedor Bogomolov (Courant Institute), and three mini-courses running from September 21 to September 23 shall be offered: "Topological methods in the study of singularities" (Anne Pichon and Walter Neumann), "The Shaferevich conjecture and the fundamental group" (Terence Napier and Mohan Ramachandran), and "Logarithmic genera and the BMY inequality" (Ryoichi Kobayashi and Adrian Langer).

A third workshop, on *Higher Teichmüller – Thurston Theory*, will be held on October 15 – 19, 2012, and is organized by Virginie Charette (Sherbrooke), William Goldman (Maryland), François Labourie (Paris-Sud) and Anna Wienhard (Princeton). Higher Teichmüller theory deals with the deformation theory of geometric structures in low dimensions. This workshop will bring together experts on a range of subjects, such as classical Teichmüller theory, Hitchin theory, Lorentzian geometry, dynamics and moduli spaces.

The following spring, a fourth workshop on J-*Holomorphic Curves in Symplectic Geometry, Topology and Dynamics* will be held April 30 – May 11, 2013, organized by Denis Auroux (Berkeley), Octav Cornea (Montréal), Yael Karshon (Toronto), François Lalonde (Montréal) and Leonid Polterovich (Chicago). The event is spread over two working weeks and will include four mini-courses and approximately thirty research lectures. The main themes reflected in the conference will be mirror symmetry and the topology of Lagrangian submanifolds, relations with dynamics and applications to symplectic geometry. The conference will benefit from the participation of Helmut Hofer (IAS) who will give an Aisenstadt lecture.

A workshop on *The Topology of* 3*-Dimensional Manifolds* shall be held May 6 – 17, 2013, organized by Michel Boileau (Toulouse), Steven Boyer (UQÀM), Marc Lackenby (Oxford) and Alan Reid (Austin). Recent advances in the understanding of the topology of 3-dimensional manifolds have led to a conjectural picture which indicates surprising restrictions on their topology. The workshop will explore the major elements of this picture and what it says about basic classification and decision problems. The main themes to be addressed are the study of finite covers and virtual properties of 3-manifolds, the study of the set of all 3-manifolds and how they relate to each other, and the algebraic and geometric properties of 3-manifold fundamental groups and their character varieties. This event will consist of introductory mini-courses held during the first week, a workshop held during the second, and a series of Aisenstadt Chair lectures to be delivered by David Gabai (Princeton) which will provide a bridge between them.

From May 26 till June 1, 2013, a workshop on *Extremal Kähler Metrics* will be held, organized by Vestislav Apostolov (UQÀM), Claudio Arezzo (Parma), Xiuxiong Chen (Madison) and Claude LeBrun (Stony Brook). The focus will be on the link between the existence of extremal Kähler metrics and algebro-geometric notions of stability of a polarized variety. Although a precise conjecture about this link has been proposed by Yau, Tian and Donaldson, experts do not expect its complete resolution for at least another decade. However, remarkable progress is being made all the time including the verification of one direction of the conjecture through the work of Chen – Tian, Donaldson, Mabuchi and Székelyhidi – Stopa. Moreover, advances continue to be made on important subcases and in the development of new techniques. The main objective of the workshop is to bring together leading experts in the separate but related fields of geometric analysis, differential geometry and algebraic geometry to focus on a specific, hard problem at the forefront of current research. A series of Aisenstadt lectures shall be given by Gang Tian (Princeton & Peking).

The next workshop is entitled *Moduli spaces and their invariants in mathematical physics.* Organized by Jacques Hurtubise (McGill), Lisa Jeffrey (Toronto) and Johannes Walcher (McGill), it will be held on June 3 – 14, 2013. Apart from their intrinsic geometric interest, moduli spaces are a natural focus for the interplay with a wide range of ideas from physics, topology, and number theory. This workshop will aim to highlight links between the great variety of topics, loosely organized around four themes. During the first week, we will work primarily on symplectic and Poisson geometry, and derived categories and their moduli. The second week will concentrate on algebraic structures, and on matrix models and integrable systems.

The thematic year will close with a workshop on *Low-Dimensional Topology after Floer*, held on July 8 – 12, 2013, and organized by Olivier Collin (UQÀM) and Hans Boden (McMaster). The conference will focus on the current impact of Floer homology methods in low-dimensional topology. In recent years, the interactions between geometric, contact and symplectic topology and the many types of Floer homologies have proved very successful in solving outstanding questions in low-dimensional topology, and there are signs that more breakthroughs will occur in the near future. This event will bring together leading experts working in these related areas, but also a special emphasis will be put on the invitation of up-and-coming researchers and graduate students, some of whom will have attended the SMS summer school on the mathematics and physics of knot homologies.

Aisenstadt Chairs:

- Fedor Bogomolov (Courant Institute), September 2012
- David Gabai (Princeton), May 2013
- Helmut Hofer (IAS), May 2013
- Gang Tian (Princeton & Peking), May 2013

2012 CRM – Fields – PIMS Prize: Stevo Todorčević

by Claude Laflamme (University of Calgary)

Stevo Todorčević is the winner of the 2012 *CRM – Fields – PIMS Prize. The prize recognizes his work in set theory and surrounding areas of mathematics.*

He is known for introducing some of the far reaching and highly original methods in this area of mathematics. For example, his method of walks on ordinals and their characteristics after almost thirty years still remains the most important tool for building mathematical structures giving correct boundaries to various classification schemes in these areas of mathematics.

He is also known for the invention of the powerful forcing-technique of models as side conditions which he used to solve some of the most difficult problems in this area such as, for example, the S-space problem or the Katetov metrization problem, the problems that have been open for over fifty years.

*However he is also known for an equally original use of forcing in solving problems not sensitive to the axioms of set theory. Typical such use is his classification result of compact sets of Baire-class-*1 *functions that completes the initial attempt of Bourgain – Fremlin – Talagrand from the* 1970*s. Another such use is his characterization of algebras supporting a strictly positive continuous measure as the optimal axiom-independent solution to an old problem of Maharam and von Neumann that emerged in the* 1930*s and* 1940*s.*

He and his collaborators were able to connect the combinatorial Ramsey-theoretic phenomena with some relatively distant areas of mathematics such as topological dynamics or descriptive set theory. Examples of this is the famous $\mathcal{G}_0$*-dichotomy of Kechris – Solecki – Todorčević or the Kechris – Pestov – Todorčević theorem showing the close relationship between the structural Ramsey theory and the representation theory of universal minimal flows in topological dynamics.*

Stevo Todorčević has received his undergraduate, Master, and Ph.D. degree from the University of Belgrade in the years 1977, 1978, *and* 1979, *respectively. He took his first academic position in* 1979 *at the Mathematical Institute in Belgrade but soon was assuming a number of prestigious fellowships such as Miller Fellowship in Berkeley* (1983 – 1985), *Membership of the Institute for Advanced Study in Princeton* (1985 – 1986) *and Ulam Research Professorship at the University of Colorado in Boulder* (1986 – 1987). *He became a Directeur de recherche at the CNRS in Paris in* 1997 *where he still holds a part time position.*

His involvement with Canada and Canadian mathematics is quite long. It has begun as early as the Summer and Fall Semesters of 1980 *which he spent at the University of Toronto, where he was holding a number of appointments throughout* 1980*s and* 1990*s. He was awarded a Canada Research Chair at the University of Toronto in the year* 2004. *He has been an organizer of a thematic program at the Fields Institute in* 2002 *and is currently organizing another such program this fall.*

His stature as one of the world's leading set theorist was recognized by two invitations to the Congress of Logic Philosophy and Methodology of Sciences (1983 *and* 1995) *and the International Congress of Mathematicians in* 1998.

I will try to explain only some of the most typical aspects of the work of Stevo Todorčević and his influence on mathematics in Canada and the rest of the world.

As indicated above the common technical power behind most of his work is grounded on a deep understanding when Ramsey-type phenomena appear in different mathematical disguises. For example, in his early work, while solving two major problems in this area of mathematics, he had to clarify a boundary to which extend the classical theorem of Ramsey can be extended in the realm of arbitrary mathematical structures that are not necessarily countable. This is on one side his classification scheme for transitive relations on the set ω_1 of countable ordinals (the minimal such uncountable structure), and on the other, the method of walks on ordinals and their characteristics that still stands as the main method for describing 'critical' mathematical structures in this context.

While trying to communicate some of the content of the first advance to a wider group of mathematicians, he isolated combinatorial dichotomies that are today standard tools in this area of mathematics. One of these dichotomies is the *P-ideal Dichotomy* saying that in the category of ideals of countable sets on an arbitrary index set S (i.e., collections $\mathcal{I}$ of countable subsets of S that are closed under taking finite unions and subsets of its elements) there are two kinds of 'critical' ideals, one, ideals $\mathcal{I}_{\max}$ that contain *all* countable subsets of an uncountable set $T \subseteq S$, and the other, ideals $\mathcal{I}_{\min}$ that contain only countable subsets of S that have a finite intersections with each of the sets S_n appearing in some fixed countable decomposition $S = \bigcup_{n<\omega} S_n$ of the index-set S. Note that the first kind of ideals $\mathcal{I}_{\max}$ are all σ-ideals, i.e., $\bigcup_{n<\omega} a_n \in \mathcal{I}_{\max}$ for all $(a_n) \subseteq \mathcal{I}_{\max}$. The second group of ideals while not σ-ideals are all *P-ideals*, a condition saying that for every sequence $(a_n) \subseteq \mathcal{I}_{\text{min}}$ there is $b \in \mathcal{I}_{\text{min}}$ such that each a_n is included in b modulo a finite set rather than simply included (a condition that can be only be required in the case of σ-ideals).

The P-ideal Dichotomy is saying that every P-ideal on S either contains one of the ideals of the form $\mathcal{I}_{\text{max}}$ or it is included in one of the ideals of the form $\mathcal{I}_{\text{min}}$.

One of the reason behind the great success of this dichotomy stems from the fact that it is accessible to a wider group of mathematicians not familiar with the original work of Todorčević. The other reason is that many mathematical structures do naturally lead to P-ideals and therefore this dichotomy has seen a number of applications not only through the work of Todorčević and his collaborators but other mathematicians as well.

One spectacular such use of this dichotomy is the recent result of Balcar, Jech and Pazák solving a sixty-year old problem of Maharam and von Neumann about fields of sets supporting strictly positive continuous outer measure. On the technical level the P-ideal dichotomy tends to transfer problems about arbitrary structures to problems about sets of reals which then can be solved by direct means. In fact, frequently the dichotomy transfers such problem to some old and well-studied problems about sets of reals. Another and perhaps more important feature of this dichotomy lies in its great potential in predicting the results that do not depend on this principle or any other additional set-theoretic assumptions.

One such remarkable application is due to Todorčević himself when he showed that a σ-complete algebra $\mathbb{B}$ supports a strictly positive continuous submeasure if and only if $\mathbb{B}$ is weakly distributive and it satisfies the σ-finite chain condition. We recall that von Neumann in his 1937 problem of the Scottish book had the countable chain condition in place of the σ-finite chain condition and was asking for a strictly positive countable additive measure. Recent result of Talagrand shows that the existence of the countably additive strictly positive measure has to be relaxed to the existence of a strictly positive continuous submeasure. It was also known that the countable chain condition has to be changed if we are to avoid using additional axioms.

Another famous dichotomy of Todorčević that is based on the same initial Ramsey-theoretic analysis is the *Open-Graph Dichotomy* saying that graphs whose vertex sets are separable metric spaces and the edge set open symmetric irreflexive subsets of their squares are either countably chromatic or have uncountable cliques. This dichotomy has an equally rich history of applications over the last thirty years but we choose to mention only the recent result by Todorčević's former student Farah showing that the Open Graph Dichotomy implies that all automorphism of the Calkin algebras are inner. This result had immediately initiated a renewed interest in connections between these two areas of mathematics especially among mathematicians working in Canada, an activity that will surely gain in its intensity in the years to come.

Walk on ordinals and their characteristic were introduced by Todorčević in the 1980s when he lifted the known classical boundaries of Sierpiński (1933) and Galvin – Shelah (1973) to an ultimate failure of the Ramsey theorem in the realm of uncountable structures.

Walk from an ordinal β to a lower ordinal α is a finite sequence $\beta = \beta_0 > \beta_1 > \cdots > \beta_k = \alpha$ such that β_{i+1} is the minimal member of C_{β_i} that is not smaller that α. Thus, the walk is defined relative to a fixed choice of sets $C_\gamma \subseteq \gamma$ that are closed and unbounded in γ.

To such a walk from β to α one can associate various characteristics $\rho_0(\alpha, \beta), \rho_1(\alpha, \beta)$, etc., usually given by recursive definitions and characteristics that in some sense resemble distance functions in this realm. This soon emerged into a deep metric theory of ordinals with application far beyond the original purpose of solving an old problem from Ramsey theory.

For example, if we concentrate on walks between countable ordinals with and appropriate choice of C_γ $(\gamma < \omega_1)$, the characteristic $\rho_1(\alpha, \beta) = \max\big\{\rho_1(\alpha, \min\big(C_\beta \setminus \alpha\big)), |C_\beta \cap \alpha|\big\}$ that identifies the maximal weight $|C_{\beta_i} \cap \alpha|$ in a walk from β to α gives us a total ordering $C = C(\rho_1)$ on ω_1 whose Cartesian square can be covered by countably many chains and, therefore, a canonical object that must appear in any classification result of this category of structures.

Deeper applications of this methods have been made either by supplementing it by the theory of oscillations of traces, also initiated by Todorčević, or by implementing this method in other areas of mathematics. For example, J. Moore, a former student of Todorčević, using the characteristic $\rho_1(\alpha, \beta)$ together with the oscillation theory of lower traces was recently able to describe a regular hereditarily Linedelöf space that is not separable solving an old and difficult problem (the L-space problem) in this area of mathematics. It should be mentioned here that the corresponding dual problem (the S-space problem) was solved by Todorčević in the piece of work already described above, the work that eventually led to the isolation of the P-ideal Dichotomy and the Open Graph Dichotomy.

Out of the applications of the second form, I mention the work of Argyros – López-Abad – Todorčević where ρ-functions are used to implementing conditional structure into norms on vector spaces, a method which together with the Gower – Maurey technique of building hereditarily indecomposable Banach spaces resulted into a remarkable example of a non-separable reflexive Banach space that has no infinite unconditional basic sequence. We recall that Gowers and Maurey originally introduced their technique in order to solve the unconditional basic sequence problem by producing a separable reflexive Banach space that is hereditarily indecomposable, a stronger condition that cannot hold in the realm of non-separable reflexive spaces.

Some of the recent work of Todorčević concentrates on connections between Ramsey theory and other areas of mathematics. As it is well-known the work of Furstenberg and Glaser from the 1970s have shown that ideas and results from these two areas of mathematics can be used not only to reprove deep Ramsey-theoretic results such as, for example, the Szemerédi theorem, the Hales – Jewett theorem, or the Hindman theorem, but also to obtain some far reaching extensions of these results.

(*continued on page* 17)

Fall 2011 Thematic Semester: Quantum Information

Quantum information science is an interdisciplinary field lying at the boundary of mathematics, computer science and physics. The main goals of the field are to understand the fundamental nature of information in a quantum mechanical world while simultaneously trying to exploit that understanding for technological gain. Montréal has been an important centre for quantum information research from the beginning; twenty five years ago, Gilles Brassard, a local researcher, invented the first protocol for exchanging secret keys exploiting quantum mechanics. About fifteen years ago, Brassard, Claude Crépeau and collaborators discovered the famous quantum teleportation protocol following a workshop here in Montréal.

The fall 2011 thematic semester on quantum information was notable not just for the variety and success of its activities, but also for its judicious application of creative chronology. Fall 2011 officially began in June with the 11th Canadian Summer School on Quantum Information and ended when Montréal hosted the 2012 edition of the prestigious Quantum Information Conference in December 2011. In between, there was a student-only conference designed to provide young researchers with a supportive environment in which to present their research, in addition to four workshops exploring quantum information from the point of view of computer science, many-body physics, communication theory and the foundations of quantum mechanics. Renato Renner of ETH Zürich and John Preskill of Caltech together spent about five weeks in Québec as the two Aisenstadt chairs, delivering seven brilliant and well-attended distinguished lectures.

The semester also benefitted from the heroically industrious participation of Charles Bennett, Aram Harrow and Steve Flammia, the three schismatic popes of the Quantum Pontiff blog (`http://dabacon.org/pontiff`). Together, they produced live transcripts for several of the thematic semester's workshops and conferences. Readers interested in browsing the bloggers' capsule summaries of the talks can follow the links provided below.

The 11th Canadian Summer School on Quantum Information (June 6 – 15, 2011)

Over the past decade, the Canadian Summer School on Quantum Information has developed into a venerable institution as the go-to summer destination for young people interested in pursuing research in the area. The School has rotated through Calgary, Montréal, Toronto, Waterloo and Vancouver over the years, so bringing it back to the Montréal area was the perfect way to kick off the thematic semester. This 11th edition was organized by University of Sherbrooke professors David Poulin, Alexandre Blais, Michel Pioro-Ladrière and Bertrand Reulet. More than 90 students attended, representing 37 universities in 21 countries. Those international students were treated not just to challenging courses, but to a quintessentially Canadian lakeside setting at the idyllic Centre de villégiature Jouvence in Québec's Parc national du Mont-Orford. The quiet setting allowed the students, ranging from the M.Sc. to

Participants in the 11th Canadian Summer School on Quantum Information

postdoctoral level, to focus on their intense program of twelve mini-courses, offered by as many world-renowned researchers. The wide-ranging list of topics introduced students to the interdisciplinary scope of quantum information science:

- Patrice Bertet (CEA Saclay), *Superconducting qubits*
- Gilles Brassard (Université de Montréal), *Communication complexity*
- Carlton M. Caves (University of New Mexico), *High precision measurements*
- Andrew Childs (University of Waterloo — IQC), *Quantum algorithms*
- Daniel Gottesman (Perimeter Institute), *Quantum error correction*
- Kurt Jacobs (University of Massachusetts Boston), *Decoherence*
- Michele Mosca (University of Waterloo – IQC), *Quantum algorithms*
- Jason Petta (Princeton University), *Spin qubits in quantum dots*
- Robert Raussendorf (University of British Columbia), *Topological fault-tolerance*
- Renato Renner (ETH Zürich), *Quantum cryptography*
- Norbert Schuch (Caltech), *Quantum many-body physics*
- Graeme Smith (IBM Research), *Quantum channels and capacities*

The organizers also took the innovative step of making the summer school a formal graduate-level course at the Université de Sherbrooke. Students received three credits for successful completion of the School, which for the first time included a challenging final exam assembled from questions supplied by the lecturers. The organizers noted the positive effects of the looming final exam on the students' focus and work ethic. The next edition of the School will take place in Waterloo in summer 2012.

The 8th Canadian Student Conference on Quantum Information (June 16 – 17, 2011)

The student conference was held in Jouvence immediately after the summer school, making it convenient to participate in both. The conference has also become something of a tradition, organized and attended exclusively by students, free from the intimidating interference of their graduate supervisors. This year, the organizers were Université de Sherbrooke Ph.D. students Guillaume Duclos-Cianci and Olivier Landon-Cardinal. The conference fulfills a real need since traditional conferences usually provide very few opportunities for exposure to junior researchers. At the student conference, every participant had the opportunity to present his or her work as either a talk or a poster. Speakers were strongly encouraged to make their presentations accessible. In the end, 70 students participated, from countries as diverse as Colombia, Korea, India, Italy, Switzerland, Poland, Russia and the United States.

The list of speakers illustrates the diversity of the participants: Khulud Almutairi (IQIS, University of Calgary), Leonardo A. Pachon (University of Toronto), Félix Beaudoin (Université de Sherbrooke), Julien Camirand Lemyre (Université de Sherbrooke), Ran Hee Choi (IQIS, University of Calgary), Sergey Filippov (MIPT, Moscow), Kent Fisher (IQC, University of Waterloo), Jan Florjanczyk (McGill University), Jose Raul Gonzalez Alonso (University of Southern California), Kyungdeock Park (IQC, University of Waterloo), Sarah Plosker (Guelph University), Anna Przysiezna (Uniwersytet Gdánski), Cyril Stark (ETH Zürich), Xiaoya Judy Wang (McGill University), Marco Zaopo (QUIT, Università di Pavia) and Lucy Liuxuan Zhang (University of Toronto).

Workshop on Quantum Computer Science (Oct. 4 – 7, 2011)

The promise of a quantum computer is not that it will run more quickly than a traditional computer. Indeed, it is quite likely that the individual logic gates of any real quantum computer will be slower than the gates in their classical counterparts. Instead, quantum computers have the potential to reduce the scaling of running time with problem size. Most famously, Peter Shor discovered in the 1990s that a quantum computer could factor integers in an amount of time polynomial in the integer's number of digits even though there is no known algorithm for traditional "classical" computers capable of doing so. Quantum computer science includes the design of quantum algorithms and the related classification of problems according to the quantum mechanical resources required to solve them, known as quantum complexity theory

The workshop, organized by Alain Tapp of the Université de Montréal and Peter Høyer of the University of Calgary, brought together 45 researchers interested in various aspects of quantum computer science, broadly interpreted. Participants presented new algorithms, like Matthias Christandl of ETH Zürich's quasipolynomial time algorithm for testing quantum separability, and refined our understanding of quantum complexity classes, as in Andrew Ducker of MIT's study of quantum computation with non-standard sources of "advice." The talks made surprising connections to physics as well. Daniel Gottesman of the Perimeter Institute explained why finding the ground state energy of even translationally invariant one-dimensional systems can be computationally intractable while Alex Arkipov of MIT explained how optical experiments in the near future should be capable of performing calculations thought to be intractable for traditional computers.

Harry Buhrman (CWI Amsterdam) supplied one of the highlights of the workshop when he gave an experimental demonstration of "garden hose complexity," an idea he and his co-authors introduced to prove no-go theorems in the area of position-based cryptography. Pumping water through a network of pipes hung from his torso, Harry calculated the value of a function by determining which of his feet got more heavily splashed by the apparatus.

Workshop on Quantum Information and Many-Body Physics (Oct. 18 – 21, 2011)

Many recent developments in the theory of quantum information have led to important insights and applications in condensed matter physics. For instance, the theory of entanglement has shed new light on the density matrix renormalization and the real space renormalization numerical methods, culminating in a deeper understanding of the strengths of the methods and applications to a wider class of problems including critical systems and systems in more than one spatial dimension. Similarly, the theory of quantum error correction has led to new theoretical models of interacting particles which exhibit topological order, an exotic phase of matter in which excitations can have non-Abelian statistics. Moreover, the study of information propagation in a system of interacting particles was used to prove the existence of an entanglement entropy area law in the ground state of systems with local interactions. The problem of finding ground states of a system composed of interacting particles was proven to be complete for the complexity class QMA, the quantum analogue of NP. These are just a few examples illustrating the connections between quantum information and condensed matter physics.

The workshop, also organized by David Poulin, brought together experts from both domains to discuss the latest results and new directions. A major emphasis was the study of numerical techniques for the simulation of many-body quantum systems on today's "classical" computers. More traditional techniques like quantum Monte Carlo and mean field approximation were compared to new quantum information-inspired algorithms like multi-scale entanglement renormalization and matrix product states. Another theme was in understanding the nature of topological order. Sergey Bravyii of IBM described an exotic three-dimensional lattice system that would store quantum mechanical information robustly in topological degrees of freedom without the need for additional error correction, a first step towards building a "quantum hard drive." Meanwhile, Spiros Michalakis of Caltech explained his proof that topological order ensures that an energy gap between the ground and excited states is robust against weak perturbations for so-called frustration-free Hamiltonians.

Steve Flammia of the Quantum Pontiff blog produced an amazingly detailed transcript of the workshop's talks at `http://tinyurl.com/7ags843`.

Workshop on Codes, Geometry and Random Structures (Oct. 24 – 26, 2011)

One of the primary concerns of quantum information theory is the design of codes for achieving communication in noisy environments, often while simultaneously achieving cryptographic objectives. The probabilistic method is often used to prove the existence of good codes and may even play a role in more explicit and efficient constructions. At the same time, many basic quantum information theoretic tasks have natural geometric interpretations that link them to a range of other application areas like compressed sensing and approximation algorithms through shared underlying mathematics. This workshop, organized by Patrick Hayden (McGill) and Aram Harrow (University of Washington), provided a forum for participants to present the latest developments in the theory of quantum communication while highlighting the range of mathematical techniques used in the area, including representation theory, asymptotic geometric analysis, random matrix theory and operator theory.

One of the most exciting talks of the workshop also happened to be the first, by Fernando Brandao of Brazil's Universidade Federal de Minas Gerais. Random unitary transformations play a role in quantum information theory analogous to that of random functions in traditional information theory. Brandao explained how to prove that composing small random unitary "gates" generates approximate polynomial unitary designs, which are analogs of k-wise independent random variables. The talk blended elements from condensed matter theory, Markov chains and computer science. Later the same day, Marius Junge of the University of Illinois explained how Shannon's information theory can be naturally re-expressed using the language of Banach spaces. The translation then lifts to the quantum, converting basic questions about quantum information theory into analogous problems in the theory of operator spaces. For some questions, notably about how badly communication above a channel's maximum capacity must fail, this translation is quite fruitful. Participants also reported remarkable progress on the design of practical error correcting codes for communicating over quantum media. Jean-Pierre Tillich of INRIA explained how to build quantum turbo codes that are excellent at reducing if not completely eliminating errors, while Joseph Renes (ETH Zürich) and Mark Wilde (McGill) presented their respective approaches to quantum polar coding. A highlight of the workshop was an hour-long moderated discussion. While the participants objected somewhat to the organizers' semi-facetious invitation to identify which areas of quantum information research had proved to be dead ends, the resulting spirited conversation confirmed that the field as a whole is vibrant and full of interesting research problems.

Simultaneously wearing his organizer's cap and his pope's mitre, Aram Harrow managed to produce a lucid summary of the talks for the Quantum Pontiff blog at `http://tinyurl.com/897qxsu`.

Workshop on Quantum Foundations in the Light of Quantum Information III (Dec. 6 – 9, 2011)

This was the third CRM workshop with this title, the first having been held in 2000, and the second (which was also a Commune) in 2002. All were organized with great flair by Gilles Brassard and Christopher Fuchs. A main goal was "to explain quantum-mechanical phenomena as inevitable consequences of information-theoretic considerations, and to derive as much of quantum mechanics as possible from this perspective." This project has proved intellectually compelling, spawning a resurgence of interest in the axiomatic exploration of mathematically represented physical theories, fed by an infusion of fresh ideas involving composite systems, information processing, and probabilistic inference, and using mathematical tools such as category theory (physical processes as morphisms) and ordered linear spaces (for representing states and measurements by compendia of probabilities).

To an extent, QFLQI III celebrated and reviewed the enormous advances that have been made since the early CRM workshops, in no small part catalyzed by them. For example, while the story is not finished, we know much more now about the "Fuchs – Brassard conjecture": that the possibility of tamper-evident secret key distribution, combined with the impossibility of bit commitment, might go a long way toward characterising quantum theory. But to an even greater extent the workshop showcased, and furthered, exciting work that is very recent or still in progress, underlining the continued vibrancy of this line of research. A major theme that emerged was the desire to integrate information-based and axiomatic approaches to quantum theory with spacetime physics, a project that is less advanced than the informational and axiomatic understanding of quantum-information-style "Alice 'n Bob" quantum theory, but clearly a focus of current activity. Some of the formal presentations are described below, but a crucial component of the workshop's success was the ample time for informal interaction, during which a lot of serious mathematics and physics was done and new collaborations were initiated, many involving student and postdoc participants.

Sharpening a seminal 2000 result of Lucien Hardy, Markus Müller, working with Lluis Masanes, showed that finite dimensional quantum theory is characterized, among finite dimensional convex theories allowing all measurements consistent with the state space, by requiring: (1) that states of composite systems are determined by the statistics and correlations of measurements on their parts ("local tomography"); (2) that continuously reversible dynamics can take any extremal ("pure") state to any other; and (3) that each system or subsystem for which the largest set of perfectly distinguishable states has a given size (the system's "information capacity"), is isomorphic to every other system with that information capacity. With Cozmin Ududec, he showed that if reversible dynamics can take any pair of perfectly distinguishable states to any other such pair, then the system has a self-dual unnormalized state space, a very strong condition, satisfied by quantum theory and physically interesting because of its connections to time-reversal symmetry and other important properties.

The Pavia group of Chiribella, D'Ariano, and Perinotti reviewed their derivation of quantum theory from assumptions including local tomography and the existence, for every mixed state of a system A, of a "purification": a state on a composite of A with another system, having the original state as its marginal A state, unique up to

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Renato Renner: Fall 2011 Aisenstadt Chair

by Patrick Hayden (McGill University)

Despite having only received his Ph.D. in 2005, Renato Renner has already had a tremendous impact on quantum information theory and its applications to both quantum cryptography and statistical physics. Recipient of medals for the best diploma and Ph.D. theses at ETH Zürich, as well as a dissertation prize from the Association for Computing Machinery, his alma mater quickly hired him as an assistant professor in their Institute for Theoretical Physics in 2007.

The ideas and techniques introduced in Renner's Ph.D. thesis have spread like wildfire through the community of quantum information researchers. Shannon's entropy plays a central role in information theory and the von Neumann entropy an analogous role in quantum information theory. When studying optimal compression rates or communication capacities, however, the Shannon and von Neumann entropies provide the right answers only when the systems being studied have a great deal of independence in their constituents. When communicating over channels with internal memory states or trying to analyze the eavesdropping strategies of a malicious adversary, the standard entropies prove to be clumsy and often inadequate tools. In his thesis, Renner introduced the quantum min-entropy and developed a formalism for analyzing it. This new entropy reduces to the von Neumann entropy in the appropriate limit of many identical and independent quantum states, but is universally applicable. The thesis then proceeded to give a new proof of the security of quantum key distribution sufficiently general to encompass most of the known protocols. More importantly, the proof yielded quantitative bounds on the security of the protocols for finite length keys; previous arguments were only valid in the limit of infinite key lengths.

Since completing his thesis, Renner has been astonishingly productive, having written over 90 articles at last count, including an amazing 21 last year, during which he was also graciously fulfilling his duties as Aisenstadt chair. That work has now comprehensively reformulated quantum information theory in the universal min-entropy formalism. One benefit of the formalism is that the theory applies without modification to real physical systems, in which the independence assumptions justifying the use of the von Neumann entropy are often violated. In a beautiful recent Nature paper, Renner and his collaborators showed that the conditional min-entropy is proportional to the amount of work required to erase the contents of one quantum memory register without modifying another. Providing an interesting twist on the second law of thermodynamics, the fact that the conditional min-entropy is negative for some entangled quantum states means that erasing information can sometimes produce work rather than consuming it.

Renner gave a series of three Aisenstadt lectures: a public lecture and two scientific talks, one each in the Quantum Computer Science and Quantum Many-Body Physics workshops. The public lecture addressed the question: "What does quantum cryptography tell us about quantum physics?" Renner started by observing that Aisenstadt, like himself, had obtained a doctorate in Zürich and then somewhat later become involved with the CRM. While Renner's supervisor was the very eminent cryptographer Ueli Maurer, Aisenstadt worked for none other than Albert Einstein. Since Renner's talk would be devoted to consequences of cryptography for the completeness of quantum mechanics, the latter a subject of great concern to the great man himself, it seemed an auspicious beginning.

John Bell's great discovery in the 1960s was that quantum mechanics made predictions incompatible with *local realism*, the paired assumptions that signals can't propagate faster than light and that objects have well-defined states prior to being measured. Experiments have since confirmed Bell's predictions, requiring that either locality or realism be false. These developments were generally thought to refute Einstein's view that a more complete theory of physics would ultimately replace quantum mechanics. However, while Bell's result ruled out a wholesale replacement of quantum mechanics by a local realistic theory, it isn't widely appreciated that the possibility of a more complete theory of physics remained open. In his talk, Renner presented joint work with Roger Colbeck (Perimeter) that finally lays that possibility to rest. Half the challenge was formulating a clear mathematical question, but they ultimately proved a more formal version of the following statement:

Theorem. *Assume that measurement statistics are correctly predicted by quantum theory and that measurement settings can be chosen freely. Then there cannot exist any extended theory that provides additional information about the outcomes of quantum measurements.*

Not surprisingly, the formalization of "providing additional information" involves Renner's beloved quantum conditional entropies. The proof of the theorem was both ingenious and elementary, allowing him to present it in its entirety during the public lecture. The connection to cryptography is that the argument was by contradiction,

(*continued on page* 11)

John Preskill: Fall 2011 Aisenstadt Chair

by Patrick Hayden (McGill University)

John Preskill is the Richard P. Feynman Professor of Theoretical Physics at Caltech, a position which speaks both to his considerable accomplishments and his playful approach to physics. Around Caltech, he is well-known for introducing physics colloquium speakers with a cleverly crafted poem. When he came to Montréal as the Aisenstadt chair, it seemed only appropriate to try and do the same for him:

I'm delighted to welcome our speaker illustrious,
A scientist brilliant and uncommon industrious,
From quarks to the cosmos, he doesn't dissemble,
He's a storehouse of knowledge, with effort assembled.

He pointed out early the need to be leery
Of the prevailing cosmological theory,
Our universe should have been full to affliction,
With magnets diverging against Gauss' prescription.

An expert on particles, fields and forces,
He juggled axions, symmetries, masses (and courses)
But somewhere along the proverbial way,
Quantum computers became his dossier.

For corruption from bit flips all the way to bosonic
He has invented new gadgets that are just the right tonic.

For Alice and Bob filtering long-estranged Eve,
He proved that entanglement provided the sieve,

And in systems exotic confined to the plane,
He found qubits tangled in quasiparticle skeins.

He's a worthy recipient of the Aisenstadt Chair,
I now entrust you to John Preskill's care.

While the poem alludes to some of his most important discoveries, some elaboration might be in order. He began his career working at the intersection of particle physics and cosmology. Early on, he observed that incorporating grand unified theories of elementary particles into cosmology predicts the widespread production of superheavy magnetic monopoles, which is in sharp conflict with observation. Resolving the conflict ultimately led to the theory of the inflationary universe.

Inspired by Peter Shor's landmark factoring paper in the mid-1990s, Preskill became interested in whether quantum computers could ever be stabilized against imperfections and environmental noise. With his student Daniel Gottesman and others, Preskill made the profound discovery that once the noise is pushed below a certain threshold value, quantum computations can be scaled up indefinitely. Contrary to expectation, longer and larger computations do not require appreciably more precise apparatus. In the absence of the threshold theorem, quantum computation would have been a theoretical curiosity without any prospect of ever becoming an engineering reality. With the theorem in hand (if not always understood), armies of experimentalists are now trying to build viable quantum computers. Over the past fifteen years, Preskill has relentlessly pursued improvements to the theory of quantum fault-tolerance. By simplifying the reasoning, improving the underlying computational building blocks and introducing more realistic models of the noise, he, as much as anyone, has helped close the gap between what experimentalists can achieve and the noise thresholds required for fault tolerance.

In another celebrated result, Preskill joined forces with Peter Shor to give a proof of the security of quantum key distribution. The goal in key distribution is for two parties to expand a very short secret into an arbitrarily long one, which can then be used as cryptographic fuel for secure encryption and many other tasks. Information-theoretically secure key distribution is impossible without invoking the laws of quantum mechanics, but in the 1980s Bennett and Brassard showed how to exploit quantum mechanics to make it work. Real-world complications like noise and loss confound their protocol and analysis, however. Université de Montréal Ph.D. student Dominic Myers found a way around those problems, but his argument was uncommonly difficult to follow and, hence, underappreciated. Preskill and Shor found a conceptually simple proof by relating the security of real-life key distribution protocols to the success of currently science-fictional entanglement distillation protocols. In cryptography, vulnerabilities stem just as often from subtle implicit assumptions as from failures of abstract reasoning, so the transparent simplicity of the Shor – Preskill proof greatly amplified its impact.

In addition to being a leading researcher, Preskill has been a prolific mentor to young scientists. He has supervised more than 40 Ph.D. students over the years, an impressive fraction of whom are now

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Quantum Information

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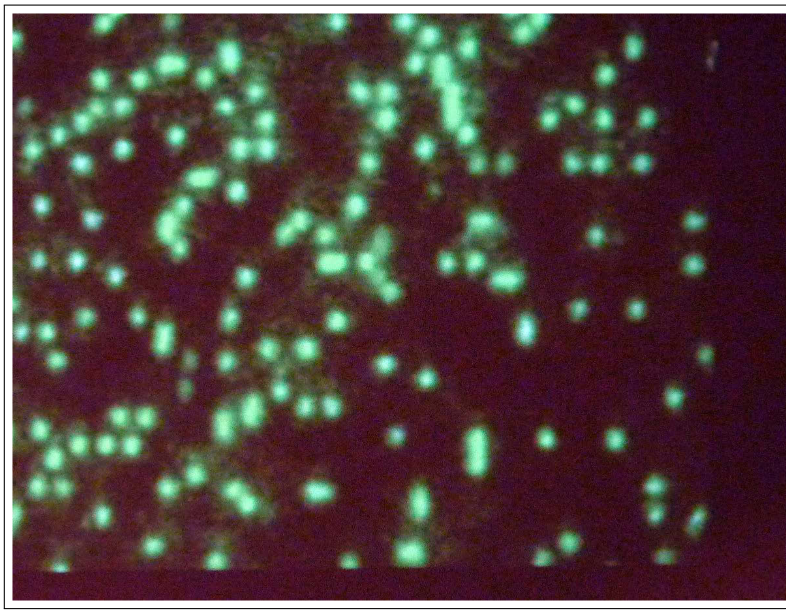
reversible transformations on the additional system, showing that given their assumptions There is One Church of the Larger Convex State Space and It is the Church of the Larger Hilbert Space. (Quantum information theorists have practiced the CLHS's rite of purification daily since ancient times.) But most of their presentations aimed to meld this operational approach with spacetime physics, with D'Ariano deriving discrete Dirac-like equations on a lattice of qubits in an approach somewhat reminiscent of work of Feynman and Jacobson, but aimed at interpreting mass and propagation speed informationally, and Chiribella and Perinotti investigating probabilistic theories where the causal structure is not fixed ahead of time. Rob Spekkens also emphasized the need to formulate quantum theory in a causally neutral way, as a theory of Bayesian inference. Howard Barnum talked about the possibility of abandoning local tomography, and thereby being able to make composites of systems whose cones of measurements are homogeneous and self-dual, as well as some information-processing notions (purification, and Schrödinger's notion of "steering" an ensemble) that imply homogeneity. Michael Westmoreland discussed the fascinating quantum-like mathematical phenomena that arise when one tries to do something like quantum theory with Z_2 instead of the complex numbers as the scalars. After some wise remarks on the nature of language and its relevance to physics, Marcus Appleby reviewed work on a beautifully simple question of pure mathematics that has seen an enormous amount of interest and effort in quantum information and foundations: Does there exist, in n-dimensional complex space with a sesquilinear inner product, an equiangular set of n^2 lines? We leave this as an exercise for the reader.

Quantum Information Processing 2012 Conference (Dec. 12 – 16, 2011)

The final event of the semester was also the largest. Quantum Information Processing (QIP) 2012, which took place at UQAM's Coeur des Sciences, had 257 registered participants from around the world. QIP is the leading conference on quantum algorithms, communication and complexity. Each year, a small handful of researchers who have made outstanding breakthroughs are invited to speak at QIP. The remaining speaking slots are filled through a competitive selection process. Since there are no proceedings, participants are free to submit work being published elsewhere, ensuring that nearly all of the year's best results get presented at QIP. Before 2012, the most recent incarnations of the conference took place in Singapore, Zürich, New Mexico and New Delhi. Further excavation of its history, however, reveals that the year 2000 incarnation of QIP was also hosted by the CRM in Montréal. This was the first time QIP has ever returned for a second time to the same city. That the quantum information community was so enthusiastic to return to Montréal is in no small part due to the exceptional support the CRM has provided over the years!

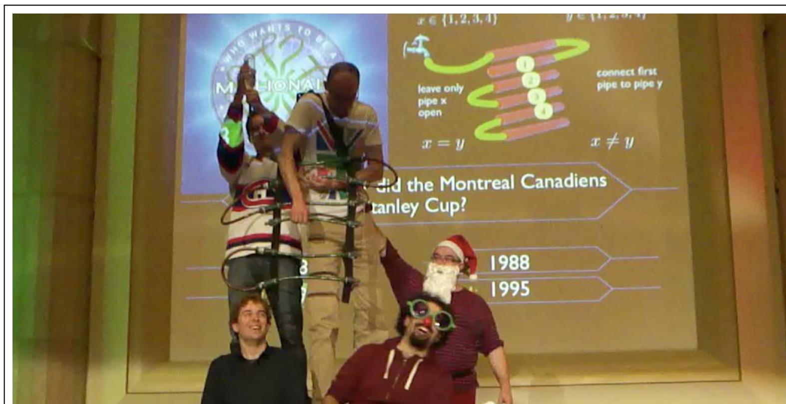
The conference's eight plenary speakers were a microcosm of the field as a whole, providing echoes of the highlights of the semester's workshops. Sergey Bravyi (IBM) and Jeongwan Haah (Caltech) spoke about three-dimensional topological quantum memories and their stability against thermal noise. Itai Arad (Hebrew University) explained how to prove an area law for one-dimensional frustration-free quantum systems that is exponentially stronger than those previously known. Aleksandrs Belovs (Waterloo) explained new characterizations of quantum query complexity, while Jérémie Roland (NEC) presented a powerful new quantum algorithmic tool called quantum rejection sampling. Meanwhile, Eric Chitambar (Michigan) disposed of a longstanding problem in quantum information theory by showing that the set of local operations with classical communication (LOCC) is not closed. Sandu Popescu (Bristol) treated the audience to a beautiful discussion in the foundations of thermodynamics about how to construct the smallest possible thermal machines.



Still from Markus Greiner's movie of individual atoms hopping in an optical lattice

Finally, Markus Greiner (Harvard) brought everyone back to reality by describing his experiments on lattices of ultracold atoms, which would have sounded like science fiction not very long ago.

A favourite QIP tradition is the "rump session," an evening devoted to short talks of dubious seriousness. In recent years, conference chair Louis Salvail had acted as rump session impresario, patrolling the stage with gong or hockey stick to chase off long-winded speakers.



Claude Crépeau, dressed as Santa Claus, presides over a game show adaptation of garden hose complexity during the QIP rump session

One advantage of being conference chair is the ability to delegate, however, so Louis passed the job on to McGill's Claude Crépeau. Seasonally stylish in a costume suggesting Saint Nick's pyjamas, Claude presided over an evening of inspired hilarity. Mathematical instruction was provided by the soon-to-syndicated "Norm on norms" before Todd Brun declaimed his poem "'Twas the night before conference". Louis Salvail made a guest appearance to advertise the Quantarctica 2012 conference, emphasizing that it would devote sessions to polar codes and ultracold atoms. A more elaborate demonstration of garden hose complexity as a dunk tank game show led to some good-natured sogginess. Many others contributed creative doses of self-deprecation to round out a thoroughly enjoyable evening.

The Quantum Pontiff bloggers, of course, covered the whole conference. Their report can be found at `http://dabacon.org/pontiff/?p=5865`. QIP 2013 will take place at Tsinghua University in Beijing.

Renato Renner

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making use of an experiment inspired by previously proposed quantum key distribution protocols.

For his talk in the Quantum Computer Science workshop, Renner spoke about "Free randomness amplification," another project with Roger Colbeck. The mathematical concept of free randomness is actually a way of formalizing, through the use of random variables located in spacetime, the words "chosen freely" used in the theorem above. In his lecture, he showed how a weak source of free randomness could be processed and upgraded into a form sufficiently good to be used in the theorem or in cryptographic applications. Crucially, the argument doesn't assume the validity of quantum mechanics, just the impossibility of signalling faster than light. Finally, changing gears somewhat, Renner devoted his final lecture, in the Quantum Information in Many-Body Physics workshop, to "An information-theoretic view on thermalization." The talk sketched some of the implications of our rapidly advancing understanding of quantum information for thermodynamics, including the remarkable discovery mentioned earlier in this article that work can sometimes be extracted when erasing parts of entangled quantum states.

As a young researcher, Renner still has many productive decades ahead of him, towards the conclusion of which epic poems recounting his exploits will no doubt be written and recited in front of blackboards the world over. In the meantime, we'll all have to make do with a limerick:

To mix information with physics,
Take your lead from Renato in Zürich.
    Embrace channels with memory
    And eavesdroppers sundry,
Until those min-entropies click.

John Preskill

(*continued from page* 9)

leading researchers themselves. His lecture notes on quantum computation form one of the standard references on the subject. And in 2000, he founded the Institute for Quantum Information (IQI) at Caltech which for many years was the undisputed theoretical hub of the subject. (Today it is just the disputed hub.) Many of Canada's leading young researchers in quantum information spent time as postdoctoral fellows at the IQI, including Andrew Childs (Waterloo), Debbie Leung (Waterloo), Ashwin Nayak (Waterloo), David Poulin (Sherbrooke), and Robert Raussendorf (UBC). (I was lucky to spend three years there myself.)

As the Aisenstadt chair, Preskill delivered a series of four lectures: a public lecture, physics colloquia at McGill and Sherbrooke, as well as a research talk in the Codes, Geometry and Random Structures workshop. The public lecture, entitled "Putting weirdness to work: quantum information science," gave a high-level introduction to quantum algorithms, key distribution and fault-tolerance, ending with a report on the current experimental state of the art. The physics colloquia were devoted to "Battling decoherence: the fault-tolerant quantum computer." In each, Preskill managed to explain the crucial ideas behind quantum fault-tolerance in just under an hour. Starting with an explanation of how quantum error correcting codes can be used to correct continuous families of errors, he then showed how to compute with encoded qubits, ultimately building to the recursive error-suppression of full fault-tolerant quantum computation. To analyze the recursive construction, he used a particularly versatile and robust version of the threshold argument that he found with Gottesman and Aliferis in 2005. The talk ended with a quick introduction to topological quantum computation, in which the inherent stability of the topological degrees of freedom thought to be present in certain exotic materials could eliminate or, at least mitigate, the need for active quantum error correction.

Preskill's contribution to the Codes, Geometry and Random Structures workshop was a remarkable hybrid of nonlinear quantum electronics and quantum error correction to which he gave the title "Protected gates for superconducting qubits." Over the past few years, experiments building qubits out of the macroscopic degrees of freedom of superconducting circuits have made tremendous strides. The decoherence time of such qubits, for example, has been extended by several orders of magnitude. Preskill's objective was to find a way to engineer superconducting qubits that would be intrinsically fault-tolerant, just as excitations in materials with topological order are thought to be. Remarkably, he, Peter Brooks and Alexei Kitaev showed that there is a quantum error correcting code lurking inside the previously proposed superconducting "$0-\pi$" qubit. More importantly, logic gates can be fault-tolerantly applied to the encoded qubits using nothing more exotic than a tunable Josephson coupling between an LC oscillator and the qubits in question. This work arguably increases from two to three the number of known paradigms for achieving fault-tolerant quantum computation. More importantly, it hints that there are probably others awaiting invention.

The 2012 André-Aisenstadt Prize was awarded jointly to Young-Heon Kim of University of British Columbia and Marco Gualtieri of University of Toronto. Robert McCann presents here the work of Professor Kim. A similar text on the work of Professor Gualtieri will appear in the next issue of the Bulletin.

On 2012 André-Aisenstadt Prizewinner Young-Heon Kim

by Robert McCann (University of Toronto)

Professor Kim completed degrees in mathematics at the Pohang University of Science and Technology (Bachelor's), the Korea Advanced Institute of Science and Technology (Master's), and Chicago's Northwestern University (Ph.D. '05 under Ezra Getzler). His thesis dealt with the determinant of the Laplacian as a functional on the Teichmüller space of hyperbolic metrics, and related questions in spectral geometry, such as "Can one hear the shape of a drum?" As one outgrowth of this, I understand he was able to simplify the celebrated Osgoode – Phillips – Sarnak result establishing compactness of the family isospectral planar domains, by reducing part of the proof which addresses domains with boundaries to the boundaryless case.

It was as a fresh Ph.D. that Y.-H. Kim first came to Canada, to be a postdoctoral fellow in the 2005 – 2006 Fields Institute programs on Renormalization and Holomorphic Dynamics, Laminations, and Hyperbolic Geometry. He impressed a number of my colleagues sufficiently during that year to be offered a further contract for an additional two years as a postdoctoral fellow at the University of Toronto.

I was on sabbatical at the time he was hired, so first met Young-Heon when he audited a course I was teaching on Optimal Transportation in the the spring of 2006. Not long after the conclusion of the course, he expressed an interest in finding a problem in the area we could work on together. I suggested he study the Hölder continuity results of Loeper concerning optimal mappings for certain cost functions, with the hope they could be applied to a free boundary version of the problem, which I had addressed with Caffarelli for the special cost $c(x,y) = |x-y|^2/2$ on Euclidean space.

At a critical point, Loeper's argument relied on a deeper result of Trudinger and Wang. Young-Heon quickly realized that it would be a great advantage to have a direct proof of this result, both for the free boundary problem and its generalization to manifolds. Trudinger and Wang had been searching for such a proof themselves, in the course of repairing a flaw which had emerged in their work with Ma — but were only partly successful in simplifying their original argument which required heavy machinery from the theory of fully nonlinear PDE. Young-Heon impressed me enormously by producing the desired proof on his own. This subtle argument spurred our realization that there is a deep differential geometric structure underlying the theory, based on a pseudo-Riemannian metric hidden in the problem. This marked the beginning of an ongoing and fruitful collaboration for both of us.

Given Borel probability densities $(M^\pm, f^\pm)$ on two manifolds of equal dimension, the Monge – Kantorovich problem is to find the minimum expectation of a cost function $c \in C^4(M^+ \times M^-)$ among all joint measures $\gamma \geq 0$ which share the marginals of $f^+ \times f^-$. To address Monge's version of this optimal transportation problem requires studying the support of the minimising γ — which is typically an n-dimensional subset of the $2n$ dimensional product space. The questions at stake are to understand when this support will be smooth, or even the graph of a homeo- or diffeomorphism (which then pushes f^+ forward to f^-). Smoothness is largely resolved by work of Caffarelli, Delanoë, Urbas, Ma, Trudinger, Wang, and others in Euclidean domains. On Riemannian manifolds $M = M^\pm$, one has the results of Loeper, which assert a negative answer for transportation with respect to Riemannian distance squared $c = d^2/2$ unless all sectional curvatures are non-negative, and a positive answer in the case of the round sphere. Aside from making Loeper's proof self-contained, my work with Young-Heon allowed us to extend his continuity results to all submersions of the round sphere, including complex projective spaces, compact manifolds of constant curvature $\kappa \geq 0$, and in joint work with Figalli, arbitrary k-fold products of spheres, each spherical factor with its own constant curvature and dimension. In a sole-author preprint, Young-Heon also solved the natural question raised by Trudinger and Loeper: whether positive sectional curvature is sufficient for the regularity of $spt\gamma$. Young-Heon's cleverly constructed counterexample uses the Topoganov comparison theorem and a perturbed convex cone to show it is not.

Since that time, he and I have continued our joint work in several other directions as well. The first of these has been to investigate connections of the structures we discovered to pseudo-Riemannian and symplectic geometry. As an example of an unexpected link that we found, let me point out that any cost $c \in C^4(M^+ \times M^-)$ induces a symplectic structure

$$\omega = \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 c}{\partial x^i \partial y^j} (dx^i \otimes dy^j - dy^j \otimes dx^i)/2$$

on the product space $N = M^+ \times M^-$ if (and only if) our corresponding pseudo-metric

$$h = -\sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 c}{\partial x^i \partial y^j} (dx^i \otimes dy^j + dy^j \otimes dx^i)/2$$

is non-degenerate.[1] Our result states that the support of the optimizer $\gamma \geq 0$, if smooth, is Lagrangian with respect to ω and spacelike with respect to h. Conversely, if the cost satisfies the condition (A3w) required for Ma, Trudinger, Wang (and Loeper's) regularity theory, we show the graph of any diffeomorphism which is ω-Lagrangian and h-spacelike is in fact the optimal map between any two densities f^+ and f^- which it transports. Finally, we interpret (A3w) — which appears to be a technical condition in the work of Ma, Trudinger, and Wang, and whose global geometric consequences were first discovered by Loeper — as the non-negativity of the sectional curvature of every two-plane spanned by orthogonal horizontal and vertical vectors in the pseudo-Riemannian product manifold (N, h).[2] We have also understood the global geometry required for $spt\gamma$ to be the graph of a diffeomorphism as h*-geodesic convexity* of all horizontal and global fibres in the product space $N = M^+ \times M^-$.

With Micah Warren (Princeton), we discovered an unexpected connection between optimal transportation and mass-minimising currents in geometry measure theory whose full implications have yet to be plumbed. Replacing h by a conformally equivalent metric

$$\tilde{h} = \left(\frac{f^+(x) f^-(y)}{\det D^2_{x^i y^j} c} \right)^{n/2} h$$

we show the graph of a c-optimal map is $\tilde{h}$-volume maximising with respect to compactly supported spacelike perturbations. Thus such a graph has *zero* $\tilde{h}$*-mean curvature* in the $2n$-dimensional product manifold. This observation reveals a completely unsuspected link between the fully nonlinear world of Monge – Ampère type equations and the classical world of minimal surfaces!

At Christmas 2007, we began to collaborate with Alessio Figalli (now at Austin), on a long project to develop a low regularity/interior smoothness theory under the weak Ma – Trudinger – Wang condition (A3w) — as opposed to its strict variant, which yields conclusions of a very different flavor. This relied on adapting Caffarelli's convex renormalization techniques to a much more nonlinear setting, and posed some serious technical and conceptual challenges. Two years ago, under a (still degenerate) strengthening of this condition — which we call non-negative cross-curvature (B4) — we managed to obtain a crucial strict convexity and continuity result which had been absent from the literature. Although we submitted this to a top journal, Young-Heon was not satisfied with the result. It took more than a year of tremendous tenacity on Young-Heon's part to find the breakthrough which enabled us to replace (B4) with the (necessary) (A3w) condition, and to improve continuity to Hölder continuity. Until then, these two improvements seemed simply out of reach. Indeed, they now seem to rely on an easily-stated but hard-to-prove lemma quantifying the boundary behaviour of convex sets in high dimensions. We prepared this result for publication as a separate note; it is purely convex geometric and involves no optimal transportation, but relies on a recursive induction on dimension.

But I save the best for last. Also in collaboration with Figalli, we turned our attention to one of the archetypal problems in economic theory. It models a monopolist wishing to maximize her profits while transacting business with a multidimensional distribution of anonymous buyers whose preferences are known only statistically. Knowing her cost to manufacture product type y, its value $b = -c$ to potential customer type x, and the relative frequency of different customer types in the population, the monopolist wishes to know which products she should manufacture and how much to charge for each of them so as to minimize her net losses. The complete analysis of this problem in a single dimension garnered Mirrlees (1996) and Spence (2001) their Nobel prizes, but in higher dimensions only the bilinear case $b(x, y) = x \cdot y$ has been solved. Remarkably, with Figalli and myself, Young-Heon showed the non-negative cross-curvature condition (B4) described above combines with b-convexity of the manufacturer's cost to give necessary(!) and sufficient conditions for reducing the monopolist's problem to an (infinite-dimensional) convex program. This insight allowed us to deduce unexpected uniqueness and stability results, and will hopefully pave the way to much future progress — both theoretical and computational. It has appeared in *Journal of Economic Theory* — a highly respected economics journal and a rare outlet for mathematicians.

To paraphrase the famed mathematician Israel Gel′fand: "It is impossible to praise one's joint work with a collaborator without praising oneself." On the other hand, as Somerset-Maugham recognized, one writes most convincingly about the subjects one knows best. Lest the reader have the impression that all of Kim's most significant work was authored jointly with me, let me close by highlighting several novel results he developed in collaboration with postdoctoral peers.

Before taking up his present position, Kim spent the 2008 – 2009 year as a visiting member at Princeton's Institute for Advanced Study. There he met Emamuel Milman (later a postdoctoral fellow at the University of Toronto, and now tenure-stream at the Technion), with whom he found a new construction of a map which provides a contraction from the standard Gaussian to any perturbation whose density is more log-concave than the original Gaussian. The first example of such a contraction was found by Caffarelli using optimal transportation; it has proven very useful for deriving geometric inequalities and consequences in statistics and mathematical physics. The map of Kim and Milman is found by integrating the trajectories of particles diffusing under heat flow. Although it is not clear whether or not it yields the same map as Caffarelli's construction, it has many of the same applications as well as some new ones. It will be exciting to see where it leads.

The other project which should be mentioned is a joint work with Jeff Streets (UC Irvine) and Micah Warren (Princeton University), which likely also began during their time at Princeton. In this work, they tackle the question of developing an algorithm to find optimal maps on compact manifolds. With this goal in mind, they introduce a dynamical version of the problem, which uses a parabolic equation to flow from the identity map towards the desired optimal map. Assuming the regularity conditions of Ma, Trudinger and Wang are satisfied, they are able to show long time well-posedness of the dynamics and convergence to the desired result. This is an important development which I am convinced will prove as amenable to numerical computation as the precursor on which it was based: an algorithm due to Schnuerer and Smoczyk which was restricted to quadratic transportation cost on Euclidean domains.

In addition to a share of the 2012 André-Aisenstadt Prize, Young-Heon Kim was recently named the recipient of a 2012 Sloan Fellowship. I expect these awards to mark not only a celebration of his achievements so far, but the beginning of a new period of

(*continued on page* 14)

Complex Analytic and Algebraic Trends in the Geometry of Varieties

August 15 – 17, 2011

Organizers:
Karol Palka (UQÀM), Peter Russell (McGill University), and Steven Shin-Yi Lu (UQÀM).

Several long term visitors specializing in complex analytic and algebraic geometry (among them R. Gurjar, M. Koras, K. Masuda, M. Miyanishi, A. Sathaye, J. Winkelmann, D.-Q. Zhang), and three postdoctoral fellows (F. Donzelli, A. Maharana, K. Palka) were present in Montréal in summer 2011, and the conference was organized to take advantage of their presence. Several research projects resulting from this collaboration are now ongoing.

There were 15 presentations of 50 minutes duration during the conference. If one looks for a common denominator for a majority of the talks, one will find it in the study of group actions, including infinitesimal ones, on varieties, in one of a manifold of incarnations of this subject.

The talks of Zhang and Gilligan dealt with compact Kähler manifolds, the first with the study of the dynamics of automorphisms from the view point of the minimal model program and the second with the existence of (local) Kähler structure on homogeneous spaces and tube neighbourhoods of CR-solvmanifolds.

The talks of Miyanishi, Masuda, Daigle and Moser-Jauslin dealt with automorphisms of affine varieties. Miyanishi characterized derivations D on a factorial affine threefold $\mathrm{Spec}(A)$ that are locally nilpotent (and hence come from a $\mathbf{G_a}$-action) in terms of the fibration $\mathrm{Spec}(A) \to \mathrm{Spec}(\mathrm{Ker}\, D)$. Masuda proved structure theorems for affine domains that have an *algebraic* derivation, a notion that generalizes local nilpotency and corresponds to actions of more general algebraic groups. Daigle described a general construction of normal affine surfaces that admit a non-trivial $\mathbf{G_a}$-action and have trivial canonical class (as, for instance, hypersurfaces in $\mathbb{A}^3$). Moser-Jauslin described the construction of families of contractible (hence diffeomorphic to $\mathbb{C}^3$) smooth hypersurfaces in $\mathbb{C}^4$ with pairwise non-isomorphic members. A key tool here is the use of the Makar-Limanov invariant (the intersection of the kernels of all locally nilpotent derivations) of these three-folds.

Kaliman gave a survey of the present state of Andersen – Lempert theory, that is the study of the density property of the Lie algebra generated by completely integrable vector fields in the space of all holomorphic vector fields on a Stein manifold. This and the analogous question of algebraic density on an affine variety are very active areas of research at present, in particular in connection with the study of *flexible* varieties. (A smooth point on a variety is flexible if the tangent space is generated by the tangents at the point to orbits of $\mathbf{G_a}$-actions.) Donzelli in his talk studied the connection between flexibility and the Makar-Limanov and related Derksen invariant in the case of affine surfaces.

The talk of Charette was on real 3-manifolds. She described 3-manifold quotients of Euclidian 3-space by nonsolvable groups of affine transformations (discovered by Margulis) and discussed recent results on the deformation spaces of such groups.

The talks of Gurjar and Maharana dealt with aspects of the classical subject of cyclic multiple planes, that is, cylic ramified covers of the complex plane $\mathbb{C}^2$. Gurjar described a common approach to multiple planes and branched covers of the n-sphere based on Smith's theory of finite group actions on simplicial complexes. This gives in particular a new proof of Zariski's result on the irregularity of a multiple plane. Maharana gave a classification in terms of the ramification locus of the multiple planes that have logarithmic Kodaira dimension less than 2.

Koras described the almost completed classification of closed embeddings of $\mathbb{C}^*$ in $\mathbb{C}^2$. Fairly sophisticated results in the the theory of open surfaces, in particular the logarithmic BMY inequality, are used.

Sathaye in his talk raised, and largely solved, an elementary questions on extensions of a field k: If t is a root of an irreducible polynomial $f(x)$ over k, when is $k(t) = k\big(f'(t)\big)$? The question arose in connection with the Jacobian problem.

Winkelmann's talk dealt with the relation between the diophantine behaviour of a variety X defined over a number field K and the properties of the complex manifold $X(\mathbb{C})$, in particular an analogy between infinite sets of integral points over a finite extension of K and holomorphic curves. He showed that these objects have similar lifting properties with respect to principal bundles.

The subject of Zong's talk was the monodromy of torsion points of abelian varieties and the Mumford – Tate conjecture. He outlined a proof of a theorem of Serre on big monodromy of torsion points.

Young-Heon Kim

(*continued from page* 13)

reinvorgated creative and intellectual contributions by one of Canada's rising young mathematical stars.[3]

[1] Non-degeneracy is locally generic; whether it can hold globally depends on the topology of M^+ and M^-.

[2] Such two-planes are h-null, so their curvatures have no magnitude, yet can still be assigned a sign $+, 0, -$. More or less, pseudo-Riemannian geometry arises in this problem for the same reason it arises in Einstein's theory of gravity — because the questions being posed are coordinate independent, or equivalently, invariant under diffeomorphisms. However, the metric h introduced by Kim and myself has as many timelike as spacelike dimensions, as opposed to Einstein's relativity, where time is traditionally one-dimensional.

[3] For a more leisurely introduction to optimal transportation touching on many of the results described above, and precise references, let me take a moment to shamelessly plug my lecture notes with Nestor Guillen: *Five lectures on optimal transportation: geometry, regularity and applications*, on the arXiv and at `http://www.math.toronto.edu/mccann/publications`.

“Grande Conférence” of Moshe Vardi

by Benoit Larose (Concordia University and Champlain College)

The CRM’s “Grandes Conférences” series invites scientists with a gift for communicating the most exciting recent developments in mathematics to a curious general public. From cryptography and quantum computing to chaos in meteorological or financial systems, and brain imaging or revolutions in biotechnology, all of the conferences reveal the power and beauty of cutting-edge mathematical research in a language accessible to all.

On February 16, 2012, the CRM had the privilege of hosting the lecture of Professor Moshe Y. Vardi of Rice University entitled “From Aristotle to the Pentium.” Moshe Y. Vardi is the George Professor in Computational Engineering and Director of the Ken Kennedy Institute for Information Technology at Rice University. He chaired the Computer Science Department at Rice University from January 1994 till June 2002. Prior to joining Rice in 1993, he was at the IBM Almaden Research Center, where he managed the Mathematics and Related Computer Science Department. His research interests include database systems, computational-complexity theory, multi-agent systems, and design specification and verification. Vardi received his Ph.D. from the Hebrew University of Jerusalem in 1981. He is the author and co-author of about 400 articles, two books, and is the editor of several collections.

Vardi is the recipient of numerous awards, including three IBM Outstanding Innovation Awards, the 2000 Gödel Prize, the 2005 ACM Kanellakis Award for Theory and Practice, the 2006 LICS Test-of-Time Award, the 2008 ACM PODS Mendelzon Test-of-Time Award, the 2008 ACM SIGMOD Codd Innovations Award, the 2008 Blaise Pascal Medal for Computer Science by the European Academy of Sciences, the 2008 ACM Presidential Award, the 2010 CRA Distinguished Service Award, the 2010 ACM Outstanding Contribution Award, and the 2011 IEEE Computer Society Harry H. Goode Award. He holds honorary doctorates from the Universität des Saarlandes, Germany, and the Université d’Orléans, France. Vardi is an editor of several international journals, and Editor-in-Chief of the Communication of ACM. He is a Guggenheim Fellow, as well as a Fellow of the ACM, the American Association for the Advancement of Science, the Association for the Advancement of Artificial Intelligence, and the IEEE. He was elected as a member of the US National Academy of Engineering, the American Academy of Arts and Science, the European Academy of Sciences, and the Academia Europea.

M. Vardi’s lecture consisted of a broadly accessible historical outline of the connection between formal logic and computer science. We were treated to a bird’s eye view of the last 2500 years, from Epimenides’ Liar’s Paradox to the Pentium chip, interleaved with amusing and insightful quotes from such logicians as diverse as Aristotle and Lewis Carroll. Using Leibniz’s unfulfilled dream of mechanizing reasoning as a recurring theme, Professor Vardi took us through a humorous and fascinating tour of the history of formal logic, visiting such characters as Ramon Llull, George Boole and Charles Peirce. We eventually returned to Euclid, whose great text has been in use for over 2000 years, in a discussion of what Wigner referred to as mathematics’ unreasonable effectiveness, the notion of mathematical proof and the consequences of the search by mathematicians of the late 19th and early 20th century to clarify this concept. This thread was followed, from Frege’s introduction of first order logic, via the discovery of Russell’s paradox and Russell and Whitehead’s *Principia Mathematica*, to the fall, at the hands of Gödel, Church and Turing, of Hilbert’s program to consolidate the foundations of mathematics. Professor Vardi argued that it is precisely out of this quest that computer science was born; by the early 50s, computers were built around the world, based on von Neumann’s ideas, thus fulfilling Leibniz’s dream: “from reasoning, to patterns of reasoning, to logic, to computers, to computers that reason.” M. Vardi closed the lecture with a moving quote from C. Papadimitriou on the sad fate of so many logicians such as Boole, Cantor, Frege, Gödel and Turing, and a remarkably prescient quote from Leibniz on the advent of the modern computer.

The talk was a hit with the audience, and in fact members of the public were overheard saying that Moshe Vardi’s lecture was the best of the “Grande Conférence” series…The lecture was followed by a reception in the hall of the Jean-Coutu building where the participants had the chance to discuss further with the speaker.

Workshop on Balance, Boundaries and Mixing in the Climate Problem

September 28 – 30, 2011

Organizers:
David N. Straub (McGill University), K. Shafer Smith (New York University) and Peter Bartello (McGill University)

In recent years there has been a shift of emphasis in research on balanced dynamics. Whereas early research was strongly motivated by NWP and focused on the construction and analysis of balanced models, following the important work of Lorenz, Warn, Ford, Vanneste and others, the concept of a "fuzzy slow manifold" has become widely accepted; thus research has been more concerned with imbalance (generation and interactions with the balanced flow) rather than balance per se. This is especially the case in oceanography, where there is still uncertainty concerning the closing of the energy budget.

This meeting, organized by Peter Bartello and David Straub (McGill) and Shafer Smith (Courant) was part of the Climate Change and Sustainability Program. The main topics of the meeting was the modelling of submesoscale oceanographic flows, where Rossby and Froude numbers are $O(1)$ and lengthscales of $O(1\,\mathrm{km})$. Here, by contrast with the (mesoscale) eddy-permitting models that are customarily used in GCMs, small-scale eddies and gravity-wave generation are ubiquitous; simulations are typically done in domains with dimensions of $O(100\,\mathrm{km})$ and grid spacings of $O(25\,\mathrm{m})$. Talks discussed modelling strategies, instability mechanisms (inertial instability in particular), geostrophic turbulence phenomenology, and applications. The breakdown of balance received most attention, but there were a few talks on balanced small-scale phenomena in the vicinity of boundaries, i.e., surface quasi-geostrophic-like dynamics. Mixing was discussed specifically in only a few presentations, but it was implicit in most of the oceanographic talks: by contrast with atmospheric scientists, who tend to view gravity waves as a means for transporting (pseudo)momentum, oceanographers are interested in gravity waves on account of their (diapycnal) mixing properties.

Although this was not the intention of the organizers, there was very little representation from atmospheric scientists (just 2 presentations). This probably reflects a cultural difference: NWP, which is strongly synoptic-scale-centric, has been hugely influential in the development of dynamical meteorology. I found the oceanographical talks eye-opening inasmuch as they started from different premises, ones that are closer to the modern view of geophysical fluid dynamics. Jim McWilliams' joke about meteorologists and their concern with balanced dynamics went down very well with the audience; nonetheless, it would have been nice if there had been some representation of work from the perspective of large-scale balanced dynamics. Below we summarize the most interesting presentations.

Vladimir Zeitlin (LMD/ENS) talked about inertial instability in 2-layer rotating shallow water. In the first part of the talk he reviewed the classical linear stability criterion and reminded the audience that inertial instability is often called symmetric instability in the case of the along-front $k = 0$ modes. The numerical simulations, a barotropic Bickley jet with different densities in the 2 layers, were designed to elucidate the connection between inertial instability and baroclinic instability. For small Ro, the linear analysis is recovered and the barotropic mode is the most unstable mode; for large Ro, the most unstable mode becomes baroclinic. Moreover, the baroclinic instability for $k = 0$ can be identified with the symmetric inertial instability.

Jim McWilliams (UCLA) discussed a number of theoretical ideas in the context of the California undercurrent, a separating boundary current. The generation of unbalanced motion represents one way in which the energy budget of the ocean may be closed and the tendency for (barotropic) energy to be cascaded to large scales via geostrophic turbulence counteracted. Numerical simulations showed generation of mesoscale and submesocale eddies where the undercurrent separated from the shore due to bottom topography.

Patrice Klein (IFREMER) reviewed recent work on the impact of submesoscales on larger oceanic scales. Observations suggest that eddies are ubiquitous on submescoscales, but until recently they were assumed to have no effect on larger scales. Following work on surface-quasigeostrophy, the talk focused on the impact of surface dynamics on submesoscale eddies at depth. Generally SQG works well, suggesting that the ageostrophic motion is weak, departures being related to the presence of ageostrophic instability and spontaneous gravity-wave emission. The quantification of the imbalance caught my attention: it's estimated that the energy in internal gravity waves is 5 orders of magnitude smaller than that in balanced modes. At the surface small-scale ageostrophic motion can lead to departures from SQG, i.e., to vortex asymmetry and modification of the eddy forcing.

Erich Becker (Rostock) gave a very interesting talk on numerical simulations of the atmospheric energy spectrum in a mechanistic model in which heating rates are prescribed. The numerical simulations were carried out at T330 in the horizontal with 2 vertical grids, L100 (with approximately $250\,\mathrm{m}$ grid spacing in the middle atmosphere) and L30 (with $\mathrm{d}z \sim 1\mathrm{km}$). A $-5/3$ mesoscale spectrum could be obtained only in a few special cases: unphysical hyperdiffusion, anisotropic subgrid (Smagorinsky) model, and the finer vertical grid. A detailed analysis of the spectral fluxes in the upper troposphere indicated that fluxes dues to horizontal advection and adiabatic convergence are comparable, consistent with Lindborg's work on stratified turbulence. Although the tuning of the Smagorinsky scheme seemed a little contrived, the sensitivity of the mesoscale dynamics to physical parameterization would seem to hold more generally.

Jacques Vanneste (Edinburgh) spoke mostly about gravity-wave generation in shear flows. The first half was a review of work on the breakdown of balance and exponential asymptotics. The second half dealt mostly with a specific problem, a 2-d SQG-like model that has been modified to account for the exponentially-small radiation of gravity waves. An important point was that the solution for the streamfunction, which is obtained by matching to the far field, must contain an exponentially increasing component as well as an exponentially decaying one. In previous work on SQG this has not always been done.

Balu Nadiga (Los Alamos) gave a very interesting talk on energy fluxes in geostrophic turbulence. The starting point was recent work by Scott and Wang claiming, in contrast to the well-known picture due to Salmon, that there is an inverse cascade of baroclinic energy. This was investigated using the Princeton Ocean Model and LES-type diagnostics, yielding energy fluxes across scales. The main message was that the situation is more complicated than Scott and Wang implied. While Salmon's picture generally holds, an inverse baroclinic cascade can be induced by basin geometry (the so-called beta-flux) and topographic interactions.

Ian Grooms (Courant) spoke about weakly stratified rotating turbulence. Although the title is suggestive of rotation-dominated, large-scale PG or SG dynamics, he only considered the wave modes defined by normal modes. The claim, which is completely standard, is that these fast modes, which are obtained from a linear stability analysis (strictly valid only for Rossby and Froude numbers going to 0), include a balanced component. He demonstrated this by looking at various linear measures of imbalance, e.g. the mismatch between the Coriolis, buoyancy and pressure-gradient terms, for Bousssinesq as well as as limit equations for rotating convection.

Leif Thomas (Stanford) discussed the relevance of symmetric instability as a possible mechanism for the transfer of energy from balanced to unbalanced modes. In particular, he analyzed a simple model including stratification and strain. Most of us were surprised at the attention devoted to symmetric instability and imbalance; there wasn't much interest in the subject several years ago. However, recent observations indicate that this mechanism may be relevant to the ocean.

Eric d'Asaro (Washington) showed Lagrangian-float observations of shallow fronts. The main result was that there is evidence of symmetric instability where there is negative PV and, furthermore, that models do not show glaring inconsistencies with this.

Geoff Vallis (Princeton) gave two talks for the price of one. In the first half, he examined the contribution of realistic bottom topography to the kinetic energy budget, more specifically, to avoiding inertial runaway; the tentative conclusion is that the resulting unbalanced flow may be small. In the second half, he examined an idealized model of the meridional overturning circulation and compared it to GCM simulations.

Alexander Bihlo (CRM) presented a poster on invariant parameterization schemes. The idea was to use Lie-group methods, in particular the differential invariants, to derive parameterization schemes that respect the continuous symmetries; it was illustrated with an application to the barotropic vorticity equation.

In all, 27 talks were given over two and a half days. These were attended by 44 participants from eight countries. They were composed of university faculty, government researchers and an encouraging number of graduate students.

Stevo Todorčević

(*continued from page* 4)

The joint work of Kechris, Pestov and Todorčević have recently revealed another such deep connection, from another area of Ramsey theory and towards (rather than from) topological dynamics. Ramsey theory of finite mathematical structures, or *Structural Ramsey Theory* how usually it is called, is an area of Ramsey theory developing over the last forty years throughout the work of numerous mathematicians who were able to identify various *Ramsey classes* of finite structures such as, for example, finite-dimensional vector spaces over finite fields (Graham – Leeb – Rothschild), or the class of finite ordered triangle-free graphs (Nešetřil – Rödl).

What Kechris – Pestov – Todorčević showed is that, if one takes the Fraïssé limit of these classes of finite structures, such kind of results become *equivalent* to a specific dynamical property of the automorphism group of the limit structure, a dynamical property commonly designated as *the extremal amenability* of the corresponding group of automorphisms, a property saying that any continuous action of the group on a compact space must have a fixed point.

Using this characterization they went further and showed how it can be used to compute *the universal minimal flows* of this kind of topological groups. For example, they show that the universal minimal flow of the group of linear isomorphisms of the countably-dimensional vector space $V_{\aleph_0}(F)$ over a finite field F is equal to its action on the space of all natural orderings of the vector space $V_{\aleph_0}(F)$.

This initiated a large scale activity for further exploration of this connection an activity that is still expanding and in which several of the former students of Todorčević are playing an important role. For example, motivated by this work, Jasiński and Sokić have made some deep contributions to the structural Ramsey theory itself, while Nguyen Van Thé in a series of two joint works, one with López-Abad and other with Sauer, has shown that the unit sphere of the famous Urysohn metric space is oscillation stable, i.e., Lipschitz maps on the sphere must be ε-constant on isometric copies of the sphere itself. This deep result besides the one of Gowers about the sphere of c_0 identifies essentially the only two known example of metric structures with this strong dynamical property.

"Grande Conférence" of Gerda de Vries

by Michael C. Mackey (McGill University)

Last December 1, Montréal mathematicians were given the chance to hear Professor Gerda de Vries of the University of Alberta (http://www.math.ualberta.ca/~devries/) speak on "The language of life: When mathematics speaks to biology". A member of the Alberta Centre for Mathematical Biology (http://www.math.ualberta.ca/~mathbio/) Professor de Vries, President of the Society for Mathematical Biology and an acclaimed teacher, researcher and lecturer led all of us on a breath-taking tour of just a few of the fascinating areas currently occupying the attention of biomathematicians.

The starting point for the talk was a reminder that mathematical biology is not a new discipline in mathematics, but rather has a long and rich history. She noted that Leonhard Euler was led, in part, to his study of incompressible fluid flow by his interest in blood flow in arteries, and that Daniel Bernoulli had major interests in promoting vaccination for smallpox based on his analysis of morbidity and mortality data using mathematical models. Using the example of Bernoulli as a spring-board, she then moved into recent considerations about the spread of avian flu virus, SARS, the H1N1 virus and the West Nile virus showing how simple considerations derived from epidemiological models allow public health policy makers to decide between various action strategies.

From there she turned to a discussion of the formation of patterns in biology (think zebra stripes and giraffe spots) using the framework of Alan Turing's seminal paper of 1952 to explain simply how reaction-diffusion systems can generate spatio-temporal patterns. These problems are, of course, areas of active research in many centres around the world even today, in fields a diverse as the formation of patterns in embryogenesis and spatial structures in animal populations. These same questions also occur in the context of the formation of animal group patterns in the swarming behaviour of fish and insects.

In the hour that Professor de Vries had she could hardly scratch the surface of what modern mathematical biology is all about. Currently, any area of biology that one cares to name (ranging from molecular biology through organ physiology to clinical medicine and ecological situations) has an active and vibrant component of mathematicians working hand in hand with experimentalists to make more sense of the data collected either in the laboratory, the clinic, or the field and to offer hypotheses that can be tested based on realistic mathematical models for the phenomena of interest. When I first started out as a young researcher in this field 40 plus years ago, reading a few journals and going to the bi-annual Gordon Conference on Theoretical Biology would suffice to keep you up-to-date with what was happening. Now, in 2012 there are literally scores of journals devoted to various aspects of mathematical biology, and a bountiful number of conferences, and summer/winter schools which one can attend to be brought up to speed.

As is the case with many mathematicians, Professor de Vries is a multidimensional person with wide ranging interests not only in mathematics and biology but also in the fine arts. She is an accomplished quilter (see http://www.telusplanet.net/public/gdevries/) and gives wonderful lectures on the mathematics in her quilts that are just as fascinating as her lecture on mathematical biology was.

Le Bulletin du CRM

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Le Centre de recherches mathématiques (CRM) a vu le jour en 1969. Actuellement dirigé par François Lalonde, il a pour objectif de servir de centre national pour la recherche fondamentale en mathématiques et leurs applications. Le personnel scientifique du CRM regroupe plus d'une centaine de membres réguliers et de boursiers postdoctoraux. De plus, le CRM accueille chaque année entre mille et mille cinq cents chercheurs du monde entier.

Le CRM coordonne des cours de cycles supérieurs et joue un rôle prépondérant (en collaboration avec l'ISM) dans la formation de jeunes chercheurs. On retrouve partout dans le monde de nombreux chercheurs ayant eu l'occasion de parfaire leur formation en recherche au CRM. Le Centre est un lieu privilégié de rencontres où tous les membres bénéficient de nombreux échanges et collaborations scientifiques.

Le CRM tient à remercier ses divers partenaires pour leur appui financier à sa mission : le Conseil de recherches en sciences naturelles et en génie du Canada, le Fonds québécois de la recherche sur la nature et les technologies, la National Science Foundation, le Clay Mathematics Institute, l'Université de Montréal, l'Université du Québec à Montréal, l'Université McGill, l'Université Concordia, l'Université Laval, l'Université d'Ottawa, l'Université de Sherbrooke, le réseau MITACS, ainsi que les fonds de dotation André-Aisenstadt et Serge-Bissonnette.

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Le Bulletin est disponible au
crm.math.ca/docs/docBul_fr.shtml.

Appel à projets

Le Centre de recherches mathématiques (CRM) vous invite à proposer des projets d'activités scientifiques. Les activités scientifiques se divisent en deux catégories : les semestres thématiques d'une durée de six mois environ, et les conférences, ateliers ou écoles d'une durée de quelques jours à 2 semaines.

Ateliers, Écoles et Conférences

Les applications pour les ateliers, écoles ou conférences doivent être reçues par le CRM un an avant la date proposée pour la tenue de l'événement. Exceptionnellement, l'échéance d'un an peut être réduite à six mois. Les demandes doivent inclure :

- Une description détaillée de l'événement. La description doit mettre l'emphase sur les buts et la pertinence de l'événement, ainsi que sur la participation des étudiants gradués et stagiaires postdoctoraux (environ 3 à 5 pages) ;
- Une liste des principaux conférenciers ;
- Les dates proposées pour la tenue de l'événement ;
- La composition du comité organisateur, avec les noms et les affiliations des membres du comité, et leurs curriculum vitæ.

Toutes les applications doivent être envoyées par courriel au directeur du CRM : `directeur@crm.umontreal.ca`.

Semestres thématiques

Les dates limites pour les propositions pour des semestres thématiques sont le 15 mars et le 15 septembre de chaque année. Les semestres thématiques ont lieu soit du 1er janvier au 31 juin (le semestre d'hiver), ou du 1er juillet au 31 décembre (le semestre d'automne). Les applications doivent être soumises au CRM au moins 18 mois avant la date du début du programme proposé. Les activités scientifiques des semestres thématiques incluent typiquement des ateliers, écoles et conférences ; des fonds sont aussi alloués pour les stagiaires postdoctoraux, les visiteurs à court et à long terme et deux chaires Aisenstadt. Toute proposition sera examinée par la direction du CRM et par le Comité scientifique international du CRM qui évalueront la qualité scientifique du projet : les fondements scientifiques, la pertinence du projet, les grandes conjectures et défis mathématiques, etc. Si le programme est accepté, les membres du comité organisateur seront en charge de l'organisation du programme thématique, en collaboration avec le directeur et le personnel du CRM qui accompagneront les organisateurs pas à pas dans l'organisation du semestre, incluant les demandes de financement externe. Le CRM héberge aussi dix laboratoires qui participent souvent à l'organisation et au financement des semestres thématiques.

Comité organisateur. Le comité organisateur comprendra des membres de la communauté mathématique canadienne et internationale. Idéalement, au moins un membre de la communauté mathématique québécoise figurera parmi les organisateurs.

Activités scientifiques. Les activités scientifiques doivent comprendre au moins trois ateliers d'une durée d'une semaine. Les organisateurs sont aussi fortement encouragés à organiser une école consacrée en priorité à la formation des étudiants gradués et des stagiaires postdoctoraux en début de programme. Entre ces périodes de concentration, les applications doivent inclure une liste des activités auxquelles participeront les visiteurs à moyen et long terme : séminaires hebdomadaires, groupes de travail, mini-cours, cours gradués, etc. Les activités de formation prévues pour les étudiants et stagiaires postdoctoraux, tels les cours préparatoires, doivent aussi être décrites.

Chaire Aisenstadt. Cette chaire permet d'accueillir dans chaque programme thématique deux ou trois mathématiciens de très grande renommée pour un séjour allant d'une semaine à un semestre. Les titulaires de la chaire Aisenstadt donnent une série de conférences sur un sujet déterminé pour son intérêt et son impact dans le cadre de la programmation thématique, dont la première, à la demande du donateur André Aisenstadt, doit être accessible à un large auditoire. Ils sont également invités à rédiger une monographie dans la série « CRM Monographs Series » diffusée par l'American Mathematical Society.

Visiteurs a long terme et stagiaires postdoctoraux. La demande doit aussi comprendre une liste préliminaire des visiteurs à moyen et long terme. Chaque semestre thématique accueillera aussi plusieurs chercheurs postdoctoraux. La durée des stages postdoctoraux associés aux semestres thématiques sera de six mois à un an.

Publications. Les organisateurs sont encouragés à présenter des projets pour publication dans les séries « CRM Monographs Series » et « CRM Proceedings & Lecture Notes » publiées conjointement avec l'American Mathematical Society.

Sources de financement. Les organisateurs sont fortement encouragés à appliquer à d'autres sources de financement pour leur programme, par exemple auprès de la National Science Foundation (NSF) ou du Clay Mathematical Institute. Les organisateurs seront pleinement épaulés par la direction et le personnel du CRM dans chacune des demandes de financement externe.

Une proposition pour un semestre thématique au CRM doit inclure :

- Le titre et une description générale du semestre thématique, ainsi que les dates proposées pour la tenue du programme. La description doit mettre clairement en évidence les buts, la qualité scientifique et la pertinence du programme. Les activités de formation pour les étudiants gradués et chercheurs postdoctoraux doivent aussi être décrites. Cette description détaillée est la partie la plus importante de la demande (environ 5 pages) ;
- Une liste des activités scientifiques proposées, et en particulier une description de chaque atelier ou école (programme scientifique, conférenciers principaux) ;
- Une liste préliminaires des visiteurs à court et à long terme ;
- Des propositions pour les candidats aux chaires Aisenstadt ;
- La composition du comité organisateur, avec les noms et les affiliations des membres du comité, et leurs curriculum vitæ.

Tous les documents doivent être envoyés par courriel au directeur du CRM : `directeur@crm.umontreal.ca`.

Call for Proposals

The CRM (Centre de recherches mathématiques) is soliciting applications for scientific activities taking place at the CRM. The proposals are divided in two categories: the thematic semesters, of a duration of about six months, and the workshops, conferences or schools, whose duration can vary from a couple of days up to two weeks.

Workshop, Schools and Conferences

The proposal for workshop, schools or conferences must be received by the CRM at least one year before the date proposed for the event. In exceptional cases, this deadline can be reduced to six months. The proposals must include:

- A description of the event. The description must emphasize the goals, the background, and the timeliness of the event, as well as the formation activities for graduate students and postdoctoral fellows (approximately 3 – 5 pages);
- A tentative list the principal invited speakers;
- Some proposed dates for the event;
- The composition of the organizing committee, with the names, affiliations and curriculum vitae of the committee members.

All the documents must be sent to the CRM director by email at: `director@crm.umontreal.ca`.

Thematic Semesters

The deadlines for proposals for thematic semesters are March 15 and September 15 of each year. The thematic semesters take place from January 1 to June 31 (for the Winter Semester), and from July 1 to December 31 (for the Fall Semester). The applications must be submitted to the CRM at least 18 months before the beginning of the semester. The scientific activities of each semester typically include workshops, conferences and schools; some funds are also allocated to support postdoctoral fellows, short and long-term visitors and two Aisenstadt chairs. All propositions will be examined by the direction of the CRM and the International Scientific Advisory Committee which will evaluate the quality of each proposal: the scientific foundations, the pertinence and timeliness of the proposal, the main mathematical challenges and conjectures, etc. If the program is accepted, the members of the organising committee will be in charge of the organization of the thematic program, in collaboration with the director and the CRM personnel which will be present at every step of the organization of the semester, including for the applications for external funding. The CRM also include ten scientific laboratories which sometimes participate actively in the organization and financing of the thematic semesters.

Organizing Committee. The organizing committee will be formed by members of the Canadian and international mathematical community. Ideally, at least one member of the organizing committee will be chosen among the local mathematical community.

Scientific Activities. The scientific activities of the thematic semester should include at least three five-days workshops. The organizers are also encouraged to organize a school aimed primarily at postdoctoral fellows and graduate students at the beginning of the theme semester. Between the concentration periods of the workshops and the schools, the thematic semester should include scientific activities involving the long-term visitors, like weekly seminars, research workshops, mini-courses, graduate courses etc. The formation activities specially aimed at graduate students and postdoctoral fellows should be described in the application.

Aisenstadt Chairs. The Chair allows to welcome in each of the thematic programs two or three world-famous mathematicians for a one-week to a one-semester stay. The recipients of the Chair give a series of conferences on set subjects, chosen because of their relevance and impact, within the thematic program, the first of which, in compliance to the donor André Aisenstadt's wish, must be accessible to a large public. They are also invited to write a monograph in the CRM Monographs Series distributed by the American Mathematical Society.

Long-Term Visitors and Postdoctoral Fellows. A tentative list of long-term visitors should be submitted with the application. Each semester will also host several postdoctoral fellows. The duration of the postdoctoral fellowships associated to the theme semester can vary from six months to a year.

Publications. The organizers are encouraged to present some projects to be published in the series "CRM Monographs Series" or "CRM Proceedings & Lecture Notes" published by the American Mathematical Society.

Funding. The organizers are strongly encouraged to apply for other sources of funding for their thematic program, for example through the National Science Foundation (NSF) or the Clay Mathematical Institute. They will receive full support from the direction of the CRM in applying to these grant agencies.

A proposal for a thematic semester at the CRM should include:

- The title and a general description of the thematic program, with the proposed dates. The description must emphasize the background, the goals, the scientific value and the timeliness of the event. The formation activities for graduate students and postdoctoral fellows should also be stressed out. This is the most important part of the proposal (approximately 5 pages);
- A list of the proposed scientific activities, in particular a description of each workshop and school (title, organizers, time frame, scientific program, main speakers);
- A preliminary list of the long-term visitors;
- Some propositions for the Aisenstadt chairs;
- The composition of the organizing committee, with the names, affiliations and curriculum vitae of the organizers.

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