Le Bulletin du CRM

www.crm.math.ca

Printemps/Spring 2008 | Volume 14 – N° 1 | Le Centre de recherches mathématiques

Thematic Program on Probabilistic Methods in Mathematical Physics

June 2, 2008 – June 13, 2009

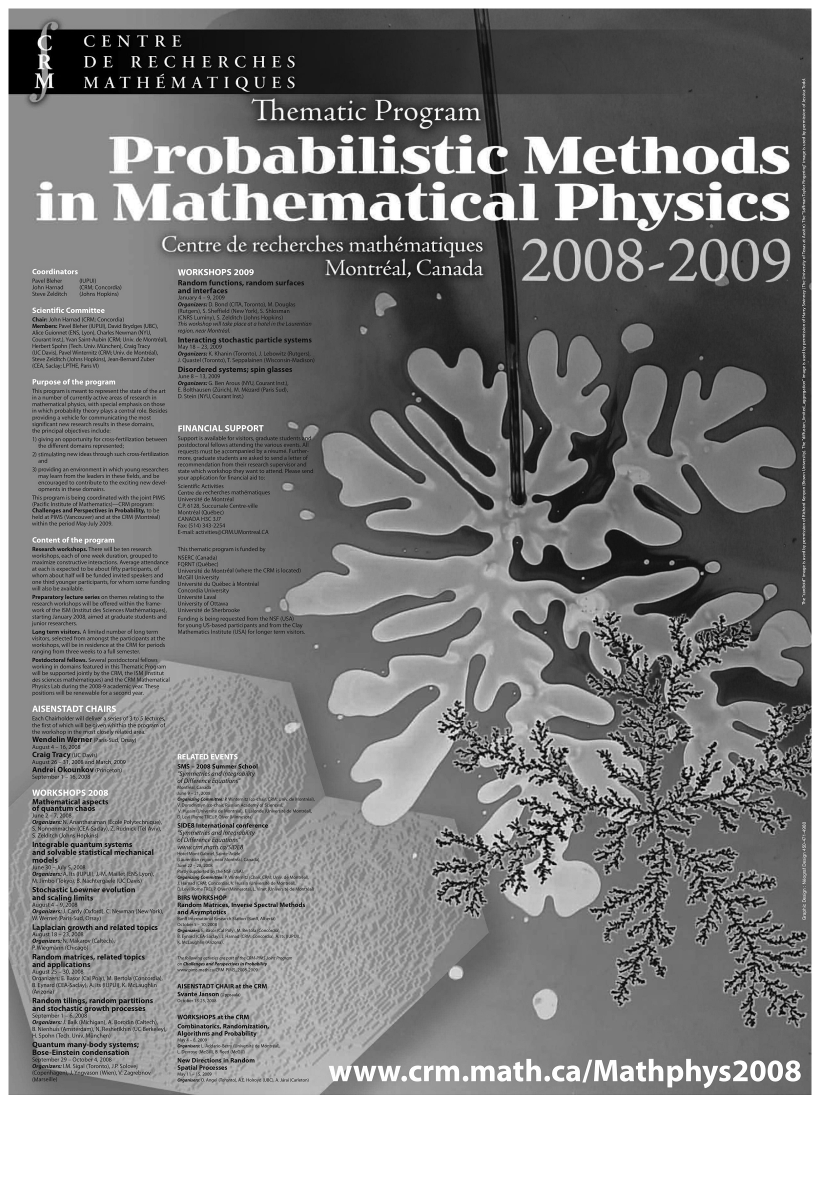
by John Harnad (Concordia University and CRM)

The coordinators for this thematic program are: Pavel Bleher (IUPUI), John Harnad (Concordia), Steve Zelditch (Johns Hopkins). They were assisted in setting the themes and choosing the organizers for the various workshops by a Scientific Committee, chaired by John Harnad, consisting of the following ten further members: Pavel Bleher (IUPUI), David Brydges (UBC), Alice Guionnet (ENS Lyon), Charles Newman (Courant Inst.), Yvan Saint-Aubin (Montréal), Herbert Spohn (TU München), Craig Tracy (UC Davis), Pavel Winternitz (Montréal), Steve Zelditch (Johns Hopkins) and Jean-Bernard Zuber (CEA, Saclay; LPTHE, Paris 6)

Randomness is present in nearly all physical measurements, and is an essential feature in the description of any physical system in which there is uncertainty in initial conditions or so many degrees of freedom that a meaningful microscopic description is only possible in terms of averaged quantities. Moreover, the probabilistic interpretation of measurement lies at the very root of the quantum description of nature. A probabilistic characterization of states, both in classical and quantum statistical mechanics, in or out of equilibrium, forms a central part of contemporary theoretical physics, from microscopic to galactic scales. It is especially important in the domain of condensed matter theory. The phases of matter, critical phenomena, localization phenomena, magnetization, conductivity, heat capacity, disordered systems, radiation and absorption, and even the very basis of quantum field theory all involve concepts derived from probability theory.

The interactions between probabilistic approaches in mathematics and physics have grown ever stronger over the past two decades. The purpose of this thematic program is to bring together mathematicians and physicists from a wide variety of fields that are linked by their common use of probability in problems of mathematical physics. It aims to represent both well-established and emerging areas of research in a series of workshops, preparatory courses, schools, and lecture series by some of the leading pioneers in a variety of domains. This will bring to the CRM a substantial portion of the international research community most active in advancing these domains, both as conference participants and long-term visitors. It will also provide an environment in which young researchers may learn from the leaders in these areas and be encouraged to contribute to the exciting ongoing developments.

Interactions and cross-fertilization between distinct fields have led to dramatic new discoveries, and suggest that much more remains to be learned. One workshop will be devoted to relatively recent developments concerning the Schramm Loewner



equation, which is seen by many as providing a promising new approach to the study of universal critical behavior in systems having scaling invariance, grouping together phenomena of apparently very distinct nature into common "universality classes" and interfacing with earlier studies based on conformal field theory. Another concerns the relatively new area of landscape statistics, which has become an important issue in string theory and cosmology. Random partitions, tilings and growth processes are relatively new areas in which remarkable interconnections have been discovered. Common elements have been found between the latter and the theory of random matrices, in which there are well-known "universality" properties that are independent of the detailed interaction dynamics involved. The domains in which these ideas have found applications range from nuclear physics to quantum field theory, quantum gravity, string theory, the growth properties of crystals, dendrites, and polymers, and even areas of pure mathematics such as number theory and enumerative topology.

One workshop will highlight the solution of one of the principal questions of quantum chaos by use of entropy methods in dynamics. Another explores the mathematical foundations of the phenomenon known as Bose-Einstein condensation, a unique state of matter which can only be attained at effectively absolute zero temperature, in which all thermal fluctuations are nearly absent, allowing the identical Bose particles to occupy a single joint ground state. This was predicted by Einstein in the early days of quantum theory, but was first observed in a laboratory only in 1995. Further workshops bear upon our understanding of nanophysics phenomena, high temperature Josephson junctions, and other essentially macroscopic quantum effects, which are accessible to exact description through the use of tools from the theory of classical and quantum integrable systems.

Interactions between mathematicians and physicists are essential to this program. Many of the workshops reflect these overlaps of interests and will give opportunities for fruitful exchanges of ideas, results and conjectures between the two groups.

In more details, the program consists of:

(1) Ten research workshops to take place at the CRM and in the vicinity, each of one week's duration. Seven of these will take place between June 2 and October 6, 2008, another in January 2009, and two more in May and June 2009. These are grouped to maximize constructive interactions between the participants. Average attendance at each is expected to be about fifty, of whom about half will be funded invited speakers and a third younger researchers and advanced students. In addition there will be a one week workshop at the Banff International Research Station in October 2008
(2) Three preparatory lecture series and an advanced study summer school on topics related to the themes of the workshops, aimed at graduate students and junior researchers,
(3) Three Aisenstadt Chair lecture series, by SIAM Polya and Wiener Prize winner Craig Tracy, and Fields medalists Andrei Okounkov and Wendelin Werner, on topics coordinated with the workshops.
(4) Twenty long-term visitors, all leading researchers within their domains, selected from amongst the organizers and participants at the workshops. Those visitors include: Pavel Bleher (IUPUI), Alexei Borodin (Caltech), Bertrand Eynard (CEA Saclay), Alexander Its (IUPUI), Greg Lawler (Chicago), Jean-Michel Maillet (ENS Lyon), Nicholas Makarov (Caltech), Barry McCoy (Stony Brook), Charles Newman (Courant Inst.), Stéphane Nonnenmacher (CEA Saclay), Alexander Orlov (Shirshov Inst. for Oceanology, Moscow), Herbert Spohn (TU München), Paul Wiegmann (Chicago), Anton Zabrodin (ITEP, Moscow), Valentin Zagrebnov (CPT, Aix-Marseille 2) as well as several younger participants.
(5) Five postdoctoral fellows working at the CRM in the domains featured.
(6) Several further related activities: the SMS Summer school, *Symmetries and Integrability of Difference Equations* (June 9–21, 2008), the SIDE 8 International Conference of the same title: *Symmetries and Integrability of Difference Equations* (June 22–28, 2008), an additional Aisenstadt Chair series, by Svante Janson (October 13–25, 2008), part of the associated thematic program on *Challenges and Perspectives in Probability* being organized jointly with the Pacific Institute for the Mathematical Sciences (PIMS), as well as two workshops organized by the PIMS scientific committee for this program in May 2009.

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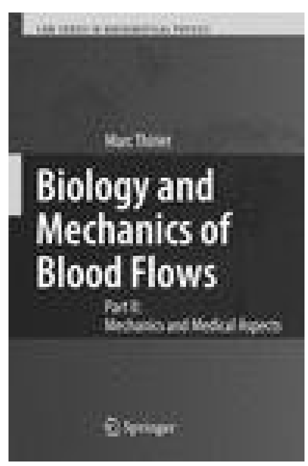
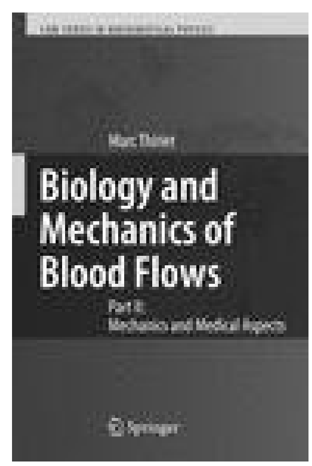




Marc Thiriet, *Biology and Mechanics of Blood Flows*
Part I: Biology; Part II: Mechanics and Medical Aspects.
CRM Series in Mathematical Physics,
Springer, New York, 2008.
ISBN: 978-0-387-74846-7; 978-0-387-74848-1

André-Aisenstadt Chairs, Fall 2007

by Tony Humphries and Michael Mackey (McGill University)

Every year, the André-Aisenstadt Chair allows the CRM to welcome, in each of the thematic programs, two or three world-famous mathematicians which are leaders in fields related to the thematic programs. The recipients of the Chair give a series of conferences on their domain of expertise, and one of them, the Aisenstadt lecture, is accessible to a large public, in compliance to the donor André Aisenstadt's wish. The two André-Aisenstadt Chair holders for the theme semester on Applied Dynamical Systems, held from June to December 2007, were John Rinzel (Center for Neural Science and Courant Institute of Mathematical Sciences, New York University) and John J. Tyson (Virginia Polytechnic Institute & State University).

John J. Tyson

John J. Tyson, was originally trained in Chemistry (B.S.) and Chemical Physics (Ph.D.). His postdoctoral training in Biophysical Chemistry at the Universität Göttingen with Manfred Eigen and in Cell Biology at Innsbruck was followed by his appointment at the Virginia Polytechnic University, Blacksburg, Virginia, where he is currently the University Distinguished Professor of Biology. John Tyson has held a number of major posts in the North American mathematical biology community, had received many national and international awards, and is the world leader in the development of quantitative mathematical models for the regulation of the eukaryote cell cycle.

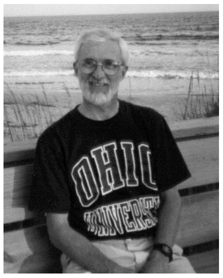
John Tyson gave three talks during his visit to the CRM in September 2007.

The first talk was a popular, didactic lecture on *Motifs and Modules in Protein Interaction Networks* delivered during the mini-course on Quantitative Biology. Tyson presented a classification scheme for reaction "motifs" (simple patterns of activation and inactivation among small numbers of proteins) and showed that these motifs have distinct and identifiable dynamical functions within regulatory networks (i.e., they are functional regulatory "modules"). Tyson then described some of the most important modules: toggle switch, sniffer, oscillator, cock-and-fire trigger, hysteresis loop, with illustrations from cell physiology.

In the Aisenstadt lecture entitled *How Do Cells Compute*?, Tyson argued that cells are "information processing systems" (like digital computers or the human brain) but the basis of information processing in cells (gene-protein interaction networks) is completely unlike silicon technology of computers or the neuronal basis of brains. Tyson showed how interacting genes and proteins can create molecular switches and clocks, and how these components are integrated within the reaction networks of cells to carry out simple yet important information processing tasks. In the climax he argued that the mathematical theory of bifurcations of vector fields can be used to catalog a small number of fundamental "signal-response" curves from which all cellular information processing must be derived.

His third lecture, delivered at the Workshop on Deconstructing Biochemical Networks, was a technical description of differential equations used for *Modeling the Eukaryotic Cell Cycle*. He showed how to use one- and two-parameter bifurcation diagrams to understand the molecular basis of cell growth and division in wild-type yeast cells and in mutants constructed by knocking out and/or over-expressing the genes encoding proteins of the regulatory system. He also presented a novel model of the very unusual regulation of mitotic cycles during early embryonic development of fruit flies.

John Rinzel



John Rinzel, Professor of Neural Science and Mathematics of the Center for Neural Science and of the Courant Institute at New York University, is interested in the biophysical mechanisms and theoretical foundations of dynamic neural computation. With a background in engineering (B.S., University of Florida, 1967) and applied mathematics (Ph.D., Courant Institute, New York University, 1973), he uses mathematical models to understand how neurons and neural circuits generate and communicate with electrical and chemical signals for physiological function. He especially relishes developing reduced, but biophysically-based, models that capture a neural system's essence. Before joining New York University in 1997, he was in the Mathematical Research Branch at the National Institute of Health (NIH) in Bethesda for nearly 25 years.

Many current research projects in the field of mathematical neuroscience find their origin in work that JohnRinzel has developed in the last decades, such as mode-locking (with collaborators Keener and Hoppensteadt), dendrites (in collaboration with Rall), waves in excitable media (with Terman, Ermentrout, Miller, etc), bursting, geometric techniques, high-speed auditory processing, and the list goes on. The influence of John Rinzel in his field is considerable, and during his years at the NIH, he has participated in the formation of large number of researchers which are now well-known leaders in the field. And perhaps more than anybody, he has promoted the opening-up of the standard neuroscience publications (*J. Neuroscience*, *J. Neurophysiology*, plus *Science* and *Nature*) to mathematical modeling in neuroscience, due in large part to the great working relationships he had established with experimentalists.

John Rinzel gave three talks during his visit at the CRM in September 2007. His Aisenstadt Lecture *Dynamics of Visual Perception* dealt with bistable perception when visualizing ambiguous scenes. The most famous example of this is the Necker cube. When viewing such scenes, our visual perception tends to flip between the valid interpretations. John Rinzel demonstrated this by subjecting the audience to ambiguous visual stimuli including various moving plaids. He also supplied the audience with 3D glasses to demonstrate the phenomenon of binocular rivalry, where each eye views different images, and perception alternates randomly between them, with a time scale of seconds. He then elegantly described the competing theories for this bi-stable perception, in which the two percepts correspond to separate firing patterns of higher level neural networks. Some theories involve deterministic switching while others rely on noise-induced switching between such firing patterns. This lecture demonstrated the state-of-the-art in the constraining of mathematical modeling frameworks by neurophysiological data.

John Rinzel's second lecture, *Timing Computations in the Auditory Brain Stem* was delivered in the Workshop on Mathematical Neuroscience. It was addressed to specialists on sound localization, a phenomenon which involves precise temporal processing by neurons in the auditory brain stem. The first neurons in the auditory pathway to receive input from both ears can distinguish interaural time differences in the submillisecond range. John Rinzel used concepts from dynamical systems and coding theory to explain the phasic firing, precise phase locking and extremely timing-sensitive coincidence detection involved in this sound localization.

John Rinzel's final presentation *Modeling the Rhythmic Dynamics of Developing Spinal Cord* was part of the Workshop on Deconstructing Biochemical Networks, and was concerned with the spontaneous rhythmic activity exhibited by many developing neural systems, where episodes of many neurons firing (say for tens of seconds) are separated by long silent phases. He described models, developed in collaboration with experimentalists at NIH, for activity patterns in the chick spinal cord, where silent phases can be very long, ten minutes or so. This behavior is network-mediated; a neuron model does not oscillate episodically if isolated. Mean-field models as well as cell-based networks of spiking neurons were used to understand the dynamics and to design experiments and then analyze results. The structural framework of the models (including bistability on the fast time scale) allows for fast/slow analysis of the emergent rhythmicity.

MIP 2007

by Odile Marcotte (CRM and GERAD)

A workshop on *Mixed Integer Programming* was held at the CRM from July 30 to August 2, 2007, and attended by 120 participants. It was sponsored by the CRM, GERAD, IBM, ILOG and Dash Optimization. The Program Committee consisted of Oktay Günlük (IBM), Matthias Köppe (Magdeburg), Andrew Miller (Wisconsin-Madison) and Jean-Philippe Richard (Purdue); the Local Committee consisted of Odile Marcotte (CRM) and Jacques Desrosiers (HEC Montréal and GERAD).

Mixed integer programming is the study of optimization problems in which some variables must take integer values while others may take fractional or real values. Mixed integer programs arise, among other fields, in portfolio selection, transport planning, design of telecommunications networks and design of cancer treatments. Recent years have seen great progress in cutting planes and other techniques for solving mixed integer programs; as a result, a group of researchers working in this area decided to organize, on a regular basis, workshops on mixed integer programming. The first three workshops took place in New York (2003), Minneapolis (2005) and Miami (2006). The program committee for MIP 2007 chose Montréal as a venue, and the workshop was attended by many members of the Montréal operations research community. For the most part, these members belong to GERAD or CIRRELT, two research centers having close links with the CRM.

The workshop featured talks pertaining to several applications. For instance, François Soumis spoke about mathematical programming problems arising in transportation planning, Daniel Bienstock about discrete models in robust portfolio optimization, Annegret Wagler about the network reconstruction problem (an important one in biology and theoretical medicine), and Eva K. Lee about optimization strategies for optimal cancer treatment design. In the latter talk, Dr. Lee mentioned the challenges faced by applied mathematicians trying to optimize medical treatments: medical technology changes (and improves) so rapidly that the mathematicians must constantly devise new algorithms to meet the challenges! George Nemhauser's talk dealt with another important model, arising from a strategic planning problem with start-time dependent costs.

As mentioned above, cutting planes play a crucial role in solving mixed integer programs, and the workshop featured eight talks on this topic (by de Farias, Louveaux, Li, Tawarmalani, Linderoth, Margot, Lodi and Smith, respectively). On the theoretical side, the workshop included a talk by Egon Balas (*Projecting systems of linear inequalities in binary variables*), a talk by Shabbir Ahmed on probabilistically constrained linear programming and a talk by Gábor Pataki on the parallel approximation problem. Laurence Wolsey spoke on the application of mixing sets to lot-sizing problems, Jacques Desrosiers on set covering and set partitioning applications, and Suvrajeet Sen on models in stochastic mixed integer programming.

The topic of symmetry breaking was addressed by Volker Kaibel, who described a procedure (called "orbitopal fixing") that improves the performance of branch-and-cut codes without explicitly adding inequalities to the model. This topic is extremely important in integer and mixed integer programming, since many natural formulations of combinatorial problems exhibit symmetries. Important work in this area was also carried out by François Margot, one of the workshop speakers. To conclude, let us mention that MIP2007 included a well-attended poster session, featuring posters of a very high quality. Some of the presentations and posters are available on the workshop web site at `http://www.crm.math.ca/MIP2007`.

2008 CRM – Fields – PIMS Prize: Allan Borodin

by Stephen Cook (University of Toronto)

Allan Borodin of the University of Toronto has been awarded the 2008 CRM – Fields – PIMS Prize. According to the citation ***Professor Borodin is a world leader in the mathematical foundations of computer science. His influence on theoretical computer science has been enormous, and its scope very broad. Jon Kleinberg, winner of the 2006 Nevanlinna Prize, writes of Borodin, "he is one of the few researchers for whom one can cite examples of impact on nearly every area of theory, and his work is characterized by a profound taste in choice of problems, and deep connections with broader issues in computer science." Allan Borodin has made fundamental contributions to many areas, including algebraic computations, resource tradeoffs, routing in interconnection networks, parallel algorithms, online algorithms, and adversarial queuing theory.***

The CRM prize lecture of Allan Borodin will take place on Friday March 28, 2008.

Borodin received his B.A. in Mathematics from Rutgers University in 1963, his M.S. in Electrical Engineering & Computer Science in 1966 from Stevens Institute of Technology, and his Ph.D. in Computer Science from Cornell University in 1969. He was a systems programmer at Bell Laboratories in New Jersey from 1963 – 1966, and a Research Fellow at Cornell from 1966 – 1969. Since 1969 he has taught with the computer science department at the University of Toronto, becoming a full professor in 1977, and chair of the department from 1980 – 1985. He has been the editor of many journals including the *SIAM Journal of Computing*, *Algorithmica*, the *Journal of Computer Algebra*, the *Journal of Computational Complexity*, and the *Journal of Applicable Algebra in Engineering, Communication and Computing*. He has held positions on, or been active in, dozens of committees and organizations, both inside and outside the University, and has held several visiting professorships internationally. In 1991 Borodin was elected a Fellow of the Royal Society of Canada.

The discipline of computer science has been an exceptionally successful blend of engineering and mathematical science with a healthy dose of human factor and aesthetic issues. Allan Borodin has made significant contributions to many diverse aspects of the discipline, with a major focus on the more mathematical areas. A common theme in his research is that he explores fundamental questions that should have well-understood explanations but seem to often defy answers to even the most basic forms of these questions. As a result Borodin has often been at the forefront of developing new models and problem formulations that have become standard frameworks for studies in computer science.

Allan Borodin

Perhaps the most basic scientific aspect of computer science is to understand the intrinsic limitations of what can and what cannot be *efficiently* computed in various models of computing with respect to various measures of complexity. This study is the heart of complexity theory. The other side of the complexity theory coin is the design and analysis of algorithms. Borodin has been involved in both sides of this coin since his first publication in 1969. In his Cornell Ph.D. thesis, Borodin studied the time complexity classes introduced by Hartmanis and Stearns and the more abstract complexity measures axiomatized by Blum. He showed that "constructible" bounding functions as used by Hartmanis and Stearns to develop complexity hierarchies are necessary in the sense that for any complexity measure (be it time, space, etc.) there are arbitrarily large gaps (where no new functions are being computed) created by non-constructible bounding functions [3]. Another thesis result (with Constable and Hopcroft) showed that time complexity classes are dense [6].

Borodin soon became more focused on the complexity of specific functions and, in particular, what we now call "algebraic complexity theory." The complexity world was basically unchartered territory at the end of the 1960s although many surprising and widely applicable results (for example, the Fast Fourier Transform and fast integer multiplication) were developed outside the confines of a formal theory. A number of results accelerated the development of complexity theory. One such result was Cook's formulation of the class NP and the identification of NP complete problems which became and still remains the main source of evidence that many common combinatorial problems cannot be solved efficiently (i.e., within polynomial time). On the other side of the coin, Strassen's surprising result that matrix multiplication can be computed within $O(n^{\log_2 7}) \approx O(n^{2.81})$ arithmetic operations showed that our common intuition and beliefs cannot be trusted (the obvious method requires n^3 multiplications). Following Strassen's dramatic result, Borodin proved a number of results helping to establish the field of algebraic complexity. Resurrecting an old question raised by Ostrovsky, Borodin showed that Horner's rule for evaluating a polynomial is uniquely optimal in being the only method that can achieve the optimal $2n$ arithmetic operations. Since (even with preconditioning) $n/2$ multiplications/divisions and n additions/subtractions are required for one polynomial evaluation for most degree n polynomials, how many operations are needed to evaluate a degree n polynomial at (say) n arbitrary points? When the evaluation points are the powers of a suitable primitive root of unity, the FFT performs these evaluations in $O(n \log n)$ operations rather than the $O(n^2)$ operations required if one evaluates at each point individually. By reduction to Strassen's fast matrix multiplication, Borodin and Munro showed that $O(n^{1.91})$ operations are sufficient [15]. Then Borodin and Moenck showed that $\Omega(n \log n)$ nonscalar multiplications and $O(nlog^2 n)$ total operations are sufficient [14], which remains the best asymptotic bound for total operations (and matched by Strassen's algebraic geometry based $\Omega(n \log n)$ lower bound for the number of nonscalar multiplications). The Strassen bound uses the Bezout Theorem on the degree of an algebraic variety to generalize the obvious fact that an nth degree polynomial requires $\log n$ multiplications (since the degree can at most double following a multiplication).

Using the FFT, two nth degree polynomials can be multiplied in $O(n)$ nonscalar multiplications and $O(n \log n)$ additions. Is there an analogue to the degree bound so as to establish lower bounds on the number of additions to compute polynomials? Borodin and Cook [4] show that the number of real roots of a polynomial is bounded by the minimal number of additions used to compute the polynomial. The Borodin and Cook lower bounds were improved by Risler using results from real algebraic geometry. Beyond these research contributions, Borodin and his first Ph.D. student Ian Munro wrote the seminal text book [16] in the area of algebraic complexity and it remained the most authoritative source for approximately 20 years.

Another area of interest for Borodin concerns parallel computation and network routing. How does parallel time complexity relate to the more standard complexity measures of time and space? Following the known results relating sequential time with uniform circuit size, Borodin showed that the space measure is directly related to uniform circuit depth, a basic measure of parallel complexity. Unlike the situation for classical sequential time studies, there are alternative models of parallel computation, including various parallel RAM models and interconnection network models. In order for an interconnection network to be able to simulate a RAM, the network must be able to simply and efficiently rout simultaneous messages. Oblivious routing schemes are simple in the sense that the path of each message is independent of the routes of other messages. Valiant showed that by obliviously routing to a random intermediate node, any permutation could be routed in time $O(d)$ on a d-dimensional hypercube. This is asymptotically optimal since d is the diameter of the network. Borodin and Hopcroft [10] showed that this use of a random intermediate node is necessary in the following strong sense: in any degree d network with n nodes, for any deterministic (i.e., nonrandomized) oblivious routing algorithm, there exists a permutation that will have a bottleneck node forcing the routing to take at least $\Omega\sqrt{n}/d^{3/2}$ time. Borodin was also the co-designer of some surprising parallel algorithms, including (with von zur Gathen and Hopcroft) [19] a randomized parallel greedy algorithm to derive a $log^2 n$ parallel time (i.e., depth of arithmetic circuit) algorithm for computing the rank of an $n \times n$ matrix, and a $\log \log n$ algorithm for merging two lists on a CREW (Concurrent Read Exclusive Write) RAM model.

The work on packet routing led to a new area of research. Packet routing can be viewed as a queuing system in which the edges of the network become the processes and one can study the queueing effects in terms the nature of the network and/or the scheduling rules used by the nodes of the network. In this setting, input requests (e.g., oblivious packet paths or requests for packet transmission along any path from source to target) are characterized more by burstiness rather than by any standard probabilistic distribution. Furthermore, the processing time (transmission of a single or a bounded number of packets along an edge) is usually considered to have a well-defined time. Borodin, Kleinberg, Raghavan, Sudan and Williamson [12] modeled this burstiness by an adversarial model and developed an area named "adversarial queuing theory." There are some natural queuing limitations on the stability (i.e., bounded queue sizes and time to complete a transmission) with the main limitation (say in oblivious routing) being that the rate of requests for an edge cannot exceed the processing rate of the edge. Borodin et al began the study as to which networks are always stable at a given rate (independent of the scheduling rule) and which scheduling rules are always stable (independent of the network). Adversarial studies of packet routing and other queueing systems has led to a number of surprising results (e.g., the instability of FIFO at any rate for certain networks as shown by Bhattacharjee and Goel).

While complexity theory has been very successful in many aspects (e.g., understanding the relation between complexity measures, establishing complexity based cryptography, utilizing hardness to develop pseudo random generators, the development of new notions of "proofs" including interactive proofs and probabilistically checkable proofs), the major limitation of the field thus far is in the inability to prove complexity impossibility results for "explicitly defined natural problems" (for example, NP search and optimization problems). More specifically, non-linear time bounds (on a sufficiently general model of computation) or space bounds greater than $\log n$ still elude us. Perhaps then the simplest barrier to break is to exhibit a problem which cannot be simultaneously computed in small time and space. Borodin and his co-authors proved in [9] the first time-space tradeoff result for comparison based sorting in what can be said to be the most general model for such a result. They considered comparison branching programs which are DAGS where nodes are labelled by comparisons "$a_i \leq a_j$?" between elements from a given "read-only" input set of n elements. In this model, edges are labelled by sequences of input elements that are being output if this edge is traversed. The complete sequence of outputs along any path defines the output of the program. In this nonuniform model (like circuits, a different program is allowed for each n), time is the length of the longest path (or expected path length if one were considering average case complexity), and space is the logarithm of the number of nodes in the DAG (i.e., the information theoretic lower bound on the memory being used). In contrast to list merging, which can be computed simultaneously in linear time and $O(\log n)$ space, Borodin et al show that the time space product $T \cdot S = \Omega(n^2)$; that is, any small space method must require significantly more time than the optimal $nlogn$ bound achievable by methods such as merge-sort. (For all space bounds $S(n)$ between $\log n$ and n, a corresponding upper bound can be obtained.)

This paper [9] was seminal and started a long and continuing research effort to derive time space bounds for natural problems in appropriate models. The sorting tradeoff was soon followed by a similar comparison branching program tradeoff for a decision problem, namely the element distinctness problem. The initial element distinctness tradeoff was by Borodin et al [8], and it was then improved by Yao. These comparison branching programs (where the algorithm does not have access to the encoding of the input elements) leaves open the possibility that the corresponding Boolean problems (e.g., say encoding integer inputs in binary) can be computed using simultaneously small time and space. This consideration led Borodin and Cook [5] to introduce the Rway branching program model, where now inputs are considered to be inputs in some range $[1, R]$, and branching program nodes are of the form "a_i =?," with up to R branches corresponding to each of the possible values of a_i. Time and space are defined as before. Borodin and Cook showed that sorting n numbers in the range $[1, n^2]$ required $T \cdot S = \Omega(n^2)$, proving a very strong tradeoff result, since the total binary encoding length of the input is only $O(n \log n)$ bits. This represents the first negative result for an explicit (polynomial time computable) Boolean problem in a completely general model, albeit not a decision problem. It took approximately 20 more years to establish negative results (of a much weaker form) for a Boolean decision problem.

In the mid 1980s, Borodin began working on the topic of online approximation algorithms which became known as competitive analysis, whereby the performance of an online algorithm (making decisions for each input as it arrives) is compared to the performance of an optimal solution with complete knowledge of the entire input. There had been a number of earlier important results concerning online algorithms for specific problems that need not necessarily be considered as online problems (for example, Graham's study of the makespan problem, Kierstead and Trotters online interval colouring, and Yao's study of online bin packing). Sleator and Tarjan proposed competitive analysis (in contrast to distributional studies) for problems which were inherently online such as paging and list accessing. Borodin, Linial and Saks [13] then proposed an abstract online problem framework called metrical task systems (MTS) which was soon followed by the k-server model of Manasse, McGeouch and Sleator. The introduction of competitive analysis for online problems and these abstract problem formulations spawned a wealth of research activity that has had an impact well beyond online problems. For example [13] provides an optimal $2n - 1$ competitive ratio bound for deterministic algorithms for any n-state MTS. It also introduced randomized algorithms in this context showing that the uniform metric system had a $2H_n \approx 2 \ln n$ randomized competitive ratio. This led the way to a randomized paging algorithm by Fiat et al and, moreover, led to interest in trying to derive randomized algorithms for general MTS and k-server problems. In this context Bartal introduced Hierarchically Separated Tree spaces (HSTs) for which $O(\log n)$ randomized algorithms exist and furthermore arbitrary metric spaces can be efficiently embedded into such HSTs. The use of HSTs has now become a standard tool in combinatorial approximation.

Beyond the seminal MTS work, Borodin was influential in a number of central results concerning online competitive analysis. Borodin et al [11] introduced a variant of competitive analysis so as to model the locality of reference exhibited by (for example) paging requests. Another landmark paper introduces "request-answer games," which provide a framework for defining most known online problems. In this very abstract setting (which, for example, includes the MTS and k-server settings), Borodin and coauthors [2] relate the power of different adversarial models for randomized online algorithms; namely, they identify the more standard oblivious adversary (as used in offline computation) where the adversary generates the input request sequence without knowledge of the algorithm's coin tosses, and adaptive adversaries where the adversary adaptively creates the input sequence by observing the coin tosses and actions of the online algorithm. For adaptive adversaries, the adversary (acting also as the "optimal benchmark") can either play the game online or play the game in hindsight as an offline player. A number of randomized algorithms were being studied relative to (not so precisely defined) adaptive adversaries. Ben David et al show that algorithms competing against online adaptive adversaries can be simulated by algorithms competing against offline adaptive adversaries which in turn can be simulated by deterministic algorithms thereby showing that randomization can only yield significantly improved competitive ratios when formulated as algorithms competing against oblivious adversaries.

Finally one of Borodin's most influential contributions to online analysis is his text [7] with former student Ran El-Yaniv. The text (published in 1998) remains the authoritative reference for this area, although many significant results have followed its publication, including a number of results addressing questions raised in the book.

Borodin has made significant contributions to a number of other aspects of algorithm analysis. One paper with Ostrovsky and Rabani [18] provides the first memory-search time results for problems (e.g., nearest neighbour and partial match search) in high-dimensional spaces, proving that for deterministic algorithms some form of exponential "curse of dimensionality" must exist for a widely studied geometric search model.

Borodin's most recent research area has been an area he has essentially been creating, namely the attempt to study the power and limitations of "simple algorithms," especially (to date) for search and optimization problems. While we equate efficient algorithms with time and/or memory efficiency, there are other important aspects to algorithm design. It is a rather remarkable fact that for over 70 years we have a well-accepted formalization (i.e., the Church – Turing thesis) for the intuitive concept of "computable function" and the associated concept of an algorithm. And if we stay within classical computing models (in contrast to say quantum computers) we have a reasonably well-accepted definition of "efficiently computable." But we often want simple understandable algorithms, at least as starting points or benchmarks for developing more sophisticated, complex algorithms. That is, we tend to use a small set of basic algorithmic paradigms as a "toolbox" for an initial (and sometimes the best known or even optimal) method for solving large classes of problems in many settings. These basic paradigms include greedy algorithms, divide and conquer, dynamic programming, local search, primal dual algorithms and IP/LP rounding. Surprisingly, although we intuitively understand what these concepts mean, rarely do we attempt any precise formulation, and a precise formulation is necessary if one is to gain any insight into the ultimate power and limitations of these methods.

The long standing and significant study and use of greedy algorithms provides a great example of an algorithmic paradigm that seems so natural and obvious that no definition seems necessary. It is hard to think of a computational area where some concept of greediness does not appear. The elegant results of Edmonds, Korte, Lovasz connecting matroids and greedoids with the optimality of "the" natural greedy algorithm for certain set systems was the starting point for a number of insightful results concerning optimal greedy algorithms. But greedy algorithms are mainly used as a heuristic or to obtain approximation results. Borodin, Nielson and Rackoff [17] introduce the priority algorithm framework as a model for "greedy-like" optimization algorithms in almost any setting. We can think of this framework as an offline extension of online algorithms. An input to a problem is a set of items (for example, jobs in a scheduling problem, vertices in a graph problem, propositional variables in a SAT problem) and a priority algorithm considers and makes decisions about items one by one but now in an order determined (in advance or adaptively) by the algorithm rather than the order given by (adversarial) nature. Of course, the trick here is formulate what orderings a "reasonable" algorithm can use. For example, it would make no sense to allow the algorithm to compute an optimal solution and a corresponding optimal order that allows the algorithm to produce the optimal solution. One approach would be to resort to complexity considerations and say that each item is chosen within some acceptable time. But that would bring us back to our current inability to prove limitations based on time complexity. Instead the priority framework relies on a simple to state concept of a local ordering. In fact, the allowable orderings are (at each iteration in the case of adaptive priority algorithms) those satisfying Arrow's IIA (independence of irrelevant attributes) axiom. Whereas in social choice theory this axiom is controversial, for greedy-like algorithms the concept allows great generality while still being amenable to analysis. And what does this have to do with greediness? In the priority framework it is not the ordering decisions that are greedy but rather (for greedy priority) it is the decisions being made for each input item that can be construed as greedy (say in the sense of "living for today") with respect to the given objective function. There are a number of results showing the limitations of such priority algorithms in different domains, starting with the initial scheduling results of Borodin, Nielson and Rackoff.

The priority framework is also the starting point for more powerful paradigms, such as some simple forms of primal dual algorithms using a reverse delete step, and simple dynamic programming and backtracking. For example, the work of Borodin and coauthors [1] shows why DPLL style backtracking algorithms cannot solve 3SAT search and has limits to approximating Max2Sat but can solve 2SAT. They also show that the form of dynamic programming used for interval scheduling and knapsack algorithms have limitations. In particular, optimal dynamic programming algorithms for weighted interval scheduling on m machines must suffer a curse of dimensionality with respect m.

This recent algorithmic design work reflects the style of an extraordinarily productive and creative career.

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(*continued on page* 22)

Étienne Ghys décrit l'effet papillon

La première Grande Conférence du CRM pour l'année 2007-2008

par Yvan Saint-Aubin (Université de Montréal)

« . . .tout s'opère, parce qu'à force de temps tout se rencontre, & que dans la libre étendue des espaces & dans la succession continue du mouvement, toute matière est remuée, toute forme donnée, toute figure imprimée ; ainsi tout se rapproche ou s'éloigne, tout s'unit ou se suit, tout se combine ou s'oppose, tout se produit ou se détruit par des forces relatives ou contraires, qui seules sont constantes, & se balançant sans se nuire, animent l'Univers & en font un théâtre de scènes toujours nouvelles, & d'objets sans cesse renaissans. » Buffon, *Histoire Naturelle des Minéraux* (1783)

La série des Grandes Conférences du CRM s'adresse au public curieux de comprendre les développements récents les plus marquants en sciences mathématiques. À raison de deux ou trois conférences annuelles, elle cherche à révéler la beauté et la puissance de la recherche mathématique de pointe dans un langage accessible à tous. Le 7 novembre dernier les GC recevaient le professeur Ghys de l'École normale supérieure de Lyon. Près de 250 personnes assistaient à son exposé.

Combien d'entre nous avons fait la une d'un quotidien important à cause d'une de nos conférences scientifiques ? Et quels quotidiens nord-américains ont-ils jamais consacré leur première page à une humble conférence de mathématiques pures ? C'est donc avec étonnement (et ravissement) que les membres de la communauté mathématique montréalaise déplièrent leur journal *Le Devoir* le 7 novembre dernier. En première page le journal claironnait que « l'effet papillon passionne les mathématiciens » et complétait un assez long article par une photo présentant à la fois Étienne Ghys, directeur de recherche au CNRS à l'École normale supérieure de Lyon, et un fragment d'attracteur de Lorentz.

Qu'est-ce qui valait une telle couverture à une Grande Conférence du CRM ? Notre communauté connaît bien le professeur Ghys, pour ses travaux en systèmes dynamiques et, particulièrement leurs aspects géométriques et topologiques, ses talents de conférencier qui ont ravi, entre autres, les participants du dernier Congrès international des mathématiciens à Madrid, et son engagement social qui se manifeste par ses excellents exposés auprès des élèves de lycées et du grand public. Mais comment toutes ses qualités, bien connues parmi nous, ont-elles traversé la barrière naturelle entre notre communauté hermétique et un représentant distingué des médias à grande distribution de Montréal ? Est-ce dû à l'effet papillon qui est devenu, dans notre société, une des indications claires des limites prédictives des mathématiques ? Ou est-ce simplement dû à la reconnaissance qu'un certain public est intéressé par l'aventure scientifique ?

Étienne Ghys

En posant la question « Est-ce qu'un battement d'ailes de papillon peut démarrer un ouragan au Texas ? », Ghys démarre son exposé avec l'attracteur de Lorentz qu'il rend concret avec un vidéo clip d'un moulin à eau de Lorenz. Mais sa description du phénomène ne suit pas un strict parcours historique. Il tisse des liens avec le passé (sensibilité aux conditions initiales telle qu'entrevue par Poincaré et Hadamard) et avec l'histoire post-Lorenz (les travaux de Smale). Il expliquera cette dépendance aux conditions initiales en présentant certains travaux qui peuvent être décrits géométriquement (les géodésiques sur les surfaces à courbures opposées selon Hadamard et son nouveau travail exposé en conférence plénière au congrès de Madrid sur la caractérisation des nœuds formés par les orbites périodiques dans le système de Lorenz). Il entreprend ensuite de caractériser plus finement le lien (sensationnaliste) entre le battement d'ailes et l'ouragan. Il rappelle ce qu'a pressenti Lorenz : « J'avance l'idée qu'au fil des années les petites perturbations ne modifient pas la fréquence d'apparition des événements tels que les ouragans : la seule chose qu'ils peuvent faire, c'est de modifier l'ordre dans lequel ces événements se produisent ». Et il explique enfin les résultats récents (W. Tucker) qui confirment l'existence de l'attracteur de Lorenz et sa robustesse (toute équation différentielle « proche » de l'équation de Lorenz aura des propriétés semblables). Ghys conclut en soulignant les deux aspects de l'effet papillon qui semblent *a priori* contradictoires. Il se manifeste par une sensibilité aux conditions initiales, car une modification mineure peut changer de manière importante le déroulement du futur. Mais aussi par une insensibilité aux conditions initiales, car la fréquence d'apparition des événements futurs, mesurée sur de grandes périodes de temps, ne dépend pas de celles-ci. L'aphorisme de Buffon, mis ci-dessus en exergue, terminait son exposé.

Quelles que soient les raisons de la couverture médiatique de cette conférence, c'était une publicité fort bienvenue. Et pour cause ! Depuis la création des Grandes Conférences, la portion de l'audience que les organisateurs reconnaissent aisément diminue ; c'est donc que nos étudiants et nos collègues, toujours présents à ces événements, ont arrêté d'en constituer la majorité. C'est maintenant le grand public qui constitue le cœur de l'audience, un public enthousiaste qui montre un intérêt marqué pour les enjeux scientifiques de notre société.

Merci à notre collègue Ghys ! Sa conférence ne peut qu'agrandir le cercle des habitués des GC du CRM !

The 2008 André-Aisenstadt Prize

Jozsef Solymosi (University of British Columbia)

The recipients of the 2008 André-Aisenstadt Prize are Jozsef Solymosi (University of British Columbia) and Jonathan Taylor (Stanford University and Université de Montréal). This year, the competition was harder than ever — the committee recognized the beauty, the impact and the splendid originality of the results of both Solymosi and Taylor. Concerning Solymosi's works, each member of the selection committee was struck by the extraordinary efficiency and elegance of his results at the cutting edge of a new field, additive combinatorics (sometimes called arithmetic combinatorics). They appreciated the simplicity and deep insight in each of his works. **See p. 12 for Taylor's article.**

The prize lectures of Jozsef Solymosi and Jonathan Taylor will take place at the CRM on May 2, 2008.

Introduction

My main field of research is in Additive Combinatorics. It is a very active area in mathematics today. This new subject brings together ideas from harmonic analysis, ergodic theory, discrete geometry, combinatorics, graph theory, group theory, probability theory and number theory. In this article I focus on three famous questions of Paul Erdős. These are my favourite problems where I have made some progress, however the questions are still open. The interested reader is invited to read more about the subject in the AMS – CRM publication *Additive Combinatorics* [12].

Sum versus Product

A central problem of this area is to characterize additive or multiplicative structures. I am especially interested in the "incompatibility" of multiplicative and additive structures. An old conjecture of Erdős and Szemerédi states that if A is a finite set of integers then the sum-set or the product-set should be large. The sum-set of A is defined as

$$A + A = \{a + b \mid a, b \in A\}$$

and the product set is

$$A \cdot A = \{ab \mid a, b \in A\}.$$

Erdős and Szemerédi conjectured that the sum-set or the product-set is almost quadratic in the size of A, i.e.,

$$\max(|A + A|, |A \cdot A|) \geq c|A|^{2-\delta}$$

for any positive δ.

Improving a result of Elekes [3] I showed in [16] that

$$|A + A|^8 |A \cdot A|^3 \geq \frac{c|A|^{14}}{\log^3 |A|},$$

and so

$$\max(|A + A|, |A \cdot A|) \geq \frac{c|A|^{14/11}}{\log^{3/11} |A|},$$

for any finite set of complex numbers, A. My bound is still very far from the conjecture.

There are variants of the sum-product problem for different fields, rings. Most notably, Bourgain, Katz, and Tao proved a nontrivial, $|A|^{1+\varepsilon}$, lower bound for the finite field case [2]. Let $A \subset \mathbb{F}_p$ and $p^\alpha \leq |A| \leq p^{1-\alpha}$. Then there is an $\varepsilon > 0$ depending on α only, such that

$$\max(|A + A|, |A \cdot A|) \geq c|A|^{1+\varepsilon}.$$

This result has important applications, not only to number theory, but to computer science, Ramsey theory, and cryptography.

Szemerédi's Theorem

Another major focus of my work in additive combinatorics is Szemerédi's theorem and variants. Szemerédi proved in 1970 that every dense subset of integers contains arbitrary long arithmetic progressions [22]. His famous theorem was reproved and extended by Furstenberg, Gowers, Rödl et al., and Tao. The most famous variant of Szemerédi's theorem is a recent result of Green and Tao. They proved that the primes contain arbitrary long arithmetic progressions [13]. Another famous extension was proved by Furstenberg and Katznelson who proved the density version of the Hales – Jewett theorem by ergodic means [9].

The Hales – Jewett theorem is a central result in Ramsey theory.

Let s be an element of the d-dimensional combinatorial cube C_k^d.

$$s = (\underbrace{0, k, 0, k-2, 2, \ldots, 1, 1}_{d}) \in [k]^d.$$

The value of the ith coordinate of s is denoted by $s|_i$. For example if s is as above, then $s|_4 = k - 2$.

A combinatorial line is given by k elements, $a_0, a_1, a_2, \ldots, a_k \in C_k^d$ such that for any coordinate j, $0 \leq j \leq d$, either $a_0|_j = a_1|_j = a_2|j = \cdots = a_k|j$ or $a_0|_j = 0$, $a_1|_j = 1$, $a_2|_j = 2, \ldots, a_k|_j = k$ and the second holds for at least one j.

The Hales – Jewett theorem states that for any natural numbers n, k there is a threshold $d_0 = d_0(n, k)$ such that if $d \geq d_0$ then for any n-coloring of C_k^d there is always a monochromatic combinatorial line.

In a joint paper with Graham [11] we gave an effective bound on d_0 for a variant of the special case $k = 3$.

Let s be an element of the d-dimensional combinatorial cube C_4^d.

$$s = (\underbrace{3,1,0,3,2,\ldots,0,1}_{d}) \in [4]^d.$$

A combinatorial triple is given by three elements, $a_0, a_1, a_2 \in C_4^d$ such that for any coordinate j, $0 \le j \le d$, $a_0|_j = a_1|_j = a_2|j$ or $a_0|_j = 0$, $a_1|_j = 1$, $a_2|j = 2$, or $a_0|_j = 3$, $a_1|_j = 2$, $a_2|j = 1$, and at least one of the last two cases holds for some coordinate j.

We proved that for any $\lfloor \log n \rfloor$-coloring of C_4^d there is always a monochromatic combinatorial triple. The best known bound for the Hales–Jewett $k = 3$ case is $\log^* n$. (The iterated logarithm of n, written $\log^* n$, is the number of times the logarithm function must be iteratively applied before the result is less than or equal to 1. The iterated logarithm is an extremely slowly-growing function, much more slowly than the $\log n$ function.)

Our proof gives some hope that maybe one can use combinatorial methods to give a quantitative (non-ergodic) proof for the density Hales–Jewett theorem, at least for the $k = 3$ case [7].

The following multidimensional generalization of Szemerédi's theorem was also proved by Furstenberg and Katznelson [6]. For every $\delta > 0$, every positive integer r and every finite subset $X \subset \mathbb{Z}^r$ there is a positive integer N such that every subset A of the grid $\{1, 2, \ldots, N\}^r$ of size at least δN^r has a subset of the form $a + dX$ for some positive integer d. Based on earlier work of Frankl and Rödl [5] I gave the first non-ergodic proof in [15] for the special case, when $X = \{(0,0), (0,1), (1,0), (1,1)\}$, proving that every dense subset of the integer grid, $\mathbb{Z}^2$, contains a square. In the same paper I noted that the following combinatorial statement would imply the general Furstenberg–Katznelson theorem. Let H be a k-uniform hypergraph on n vertices such that every edge is the edge of exactly one $(k+1)$-clique. Then the number of edges of H is $o(n^k)$. This combinatorial statement was proven by Gowers [10] and Rödl et al. [14], using the hypergraph regularity method. In [17] I showed that the combinatorial statement above implies another generalization of Szemerédi's theorem, a variant of the famous Balog–Szemerédi theorem: if a set of real numbers contains many 3-term arithmetic progressions, then the set should contain long arithmetic progressions.

Distinct Distances

My third topic has interesting connections to additive combinatorics. In their paper mentioned before, Bourgain, Katz, and Tao gave nontrivial bounds on the number of distinct distances on $\mathbb{F}_p^2$. In fact they showed that the distinct distances problem is equivalent to the sum-product problem on $\mathbb{F}_p^2$.

Erdős says in [4]: "My most striking contribution to geometry is, no doubt, my problem on the number of distinct distances."

Let $g(n)$ denote the minimum number of distinct distances determined by n points on the plane. Erdős proved that the points of a $\sqrt{n} \times \sqrt{n}$ piece of the integer lattice determine $cn/\sqrt{\log n}$ distinct distances. He conjectured that this upper-bound is asymptotically tight.

In [19], extending Székely's method, with Tóth we proved that $g(n) > cn^{6/7}$. Improving a number theoretical lemma in our paper, Katz and Tardos increased the exponent $\frac{6}{7}$ by an additional 0.007. In [18] we generalized this result to give new bounds on circle-point incidences on the plane. It would be very important to extend our investigations further to find better bounds on point-circle incidences. The general conjecture is that the number of incidences between n points and m circles cannot be much larger than the maximum number of incidences between n points and m lines.

There is a special case of the problem with an even stronger conjecture of Erdős. What is the maximum number of unit distances among n points on the plane. Erdős showed that the number of unit distances among n points can be as large as $n^{1+c/\log\log n}$, and conjectured that this was about the maximum. The best known bound is $cn^{4/3}$, far from the conjecture. There are norms, similar to the Euclidean norm where one can find $cn^{4/3}$ unit distances among n points, so any improvement, even the smallest one would be a big breakthrough. In order to improve the bound we need a better understanding of the structure of extremal circle-point arrangements.

Erdős asked the distinct distances question for higher dimensions as well and conjectured that n points in the d-dimensional Euclidean space determine at least $n^{2/d-\epsilon}$ distinct distances. With Vu we proved in [21] that the number of distinct distances is at least

$$n^{2/d-2/(d(d+2))}.$$

There is still a gap between our bound and the conjecture, however the gap is decreasing quadratically with the dimension's increase.

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(*continued on page* 14)

The 2008 André-Aisenstadt Prize

Jonathan Taylor (Stanford University and Université de Montréal)

The recipients of the 2008 André-Aisenstadt Prize are Jozsef Solymosi (University of British Columbia) and Jonathan Taylor (Stanford University and Université de Montréal). This year, the competition was harder than ever — the committee recognized the beauty, the impact and the splendid originality of the results of both Solymosi and Taylor. In Taylor's work, the committee was impressed by the exceptional breath of expertises in probability, geometry and statistics — although the committee only took into consideration the mathematical aspects of his works, this was already enough to award the Prize to Taylor with enthusiasm.

The prize lectures of Jozsef Solymosi and Jonathan Taylor will take place at the CRM on May 2, 2008.

Random fields, or random functions on a parameter space M are natural models of many phenomena: astrophysics, brain imaging, atomspheric datas. The simplest model of a random field is a Gaussian random field f which is completely specified by its mean

$$\mu(t) = \mathbb{E}\{f(t)\}, \quad t \in M$$

and a semi-positive definite covariance function $C\colon M \times M$

$$C(t,s) = \mathbb{E}\{(f(t) - \mu(t))(f(s) - \mu(s))\}.$$

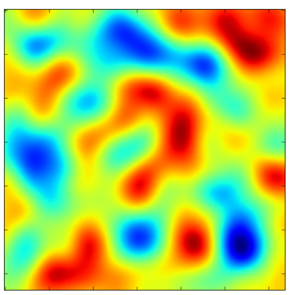


Figure 1. *Realization of an isotropic Gaussian random field*

In many statistical applications, particularly signal detection problems in brain imaging [21,22], a quantity of natural interest is the *excursion probability*

$$\mathbb{P}\Big\{\sup_{t\in M} f(t) \geq u\Big\}. \tag{1}$$

In such settings, it is often the case that $\mu \equiv 0$ and $C(t,t) = 1$, $\forall t \in M$. Finding accurate approximations to (1) is a long standing problem in the area of Gaussian processes [2, 12]. If we make the additional assumptions that T has a smooth structure as well as f (both reasonable in typical brain imaging applications) then we can in fact say much more about (1).

Critical points and suprema distributions

Obviously,

$$\begin{aligned}\mathbb{P}\Big\{\sup_{t\in M} f(t) \geq u\Big\} &= \mathbb{P}\{\exists \text{ critical point of } f \text{ in } M \text{ above the level } u\}\\ &= \mathbb{P}\{\text{number of connected components}\\ &\qquad \text{of } M \cap f^{-1}[u,+\infty) \text{ is} > 0\}. \end{aligned}\tag{2}$$

Clearly, the behaviour of critical points/values of f above the level u determines the behaviour of the supremum.

The Euler characteristic heuristic [1] approximates (1) as follows

$$\begin{aligned}\mathbb{P}\Big\{\sup_{t\in M} f(t) \geq u\Big\} &= \mathbb{P}\{\text{number of connected components}\\ &\qquad \text{of } M \cap f^{-1}[u,+\infty) > 0\}\\ &\simeq \mathbb{E}\{\chi(M \cap f^{-1}[u,+\infty))\}\end{aligned}\tag{3}$$

where χ is the Euler – Poincaré characteristic of the excursion set $M \cap f^{-1}[u,+\infty)$. The heuristic is based on the argument that at high levels, all connected components of $f^{-1}[u,+\infty)$ are *highly likely* to be simply connected, in which case $\chi(M \cap f^{-1}[u,+\infty))$ simply counts the number of connected components. Following a Poisson clumping argument, for even higher u, the probability of having at least one connected component is just the expected number of connected components which is *well approximated* by the expected Euler characteristic.

The expected Euler characteristic of the excursion set of a Gaussian process can be expressed in terms of geometric quantities related to the parameter space M and the covariance function C. Specifically, under some nondegeneracy conditions the process f determines a Riemannian metric on M

$$g(X_t, Y_t) = \mathbb{E}\{X_t f \cdot Y_t f\}. \tag{4}$$

The expected Euler characterstic for a Gaussian random field has the following form [17]

$$\mathbb{E}\{\chi(M \cap f^{-1}[u,+\infty))\} = \sum_{j=0}^{\dim(M)} \mathcal{L}_j(M)\rho_j(u)$$

where $\mathcal{L}_j(M)$ are the total Lipschitz – Killing curvatures of M [5, 14, 15, 20] and $\rho_0(u) = (2\pi)^{-1/2}\int_u^\infty e^{-x^2/2}\,dx$, $\rho_j(u) = (2\pi)^{-(j+1)/2}H_{j-1}(u)e^{-u^2/2}$, $j \geq 1$ where H_j are the Hermite polynomials

$$H_j(u) = (-1)^j e^{u^2/2}\frac{\partial^j}{\partial u^j}e^{-u^2/2}.$$

The expectation above can be computed using Morse's theorem [10, 11] and the Rice – Kac formula [6]. For locally or infinitesimally convex sets, such as manifolds with smooth boundaries, the critical points in Morse's theorem are *extended outward* critical points of f and the difference between (1) and the final display of (3) is bounded by

$$\begin{aligned}&\mathbb{E}\{\#\text{ extended outward critical points above}\\ &\qquad \text{the level } u \text{ that are not the global supremum}\}\end{aligned}\tag{5}$$

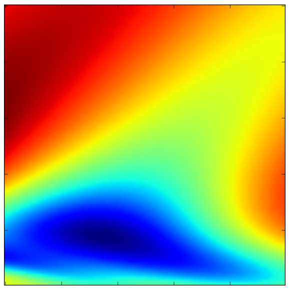


Figure 2. *A realization of a nonstationary Gaussian random field on* $[0,1]^2$. *The Riemannian metric* (4) *describes the local smoothness of the process, which influences the expected Euler characteristic and supremum distribution of* f.

The expectation above can be computed using Morse's theorem [10, 11] and the Rice – Kac formula [6]. For locally or infinitesimally convex sets, such as manifolds with smooth boundaries, the critical points in Morse's theorem are *extended outward* critical points of f and the difference between (1) and the final display of (3) is bounded by

$$\mathbb{E}\{\text{\# extended outward critical points above the level } u \text{ that are not the global supremum}\} \quad (6)$$

Theorem 1 (Accuracy of the EC heuristic, [19])**.** Suppose f is a centered, unit variance Gaussian random field on a manifold with smooth boundary M. Then, (6) is exponentially smaller then (1), and its exponential behaviour is characterized by a certain *critical variance* of the process f.

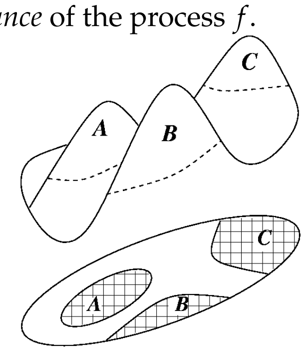


Figure 3. *The Euler characteristic of the excursion of a function on M is related to its critical points. Here, there are local maxima in each component A, B, C. With B and C having extended outward local maxima along the boundary of M.*

Integral geometry

The representation of the expected Euler characteristic through critical points is useful both for computations and deriving bounds on the accuracy of the EC heuristic.

An alternate approach is through integral geometry, specifically through the so-called Kinematic Fundamental Formula (KFF) [7, 9, 14]. Let ν_n be a Haar measure on the G_n, the group of rigid motions on $\mathbb{R}^n$ and $M_1, M_2 \subset \mathbb{R}^n$ two manifolds. The KFF reads

$$\int_{G_n} \mathcal{L}_j(M_1 \cap gM_2)\, d\nu_n(g) = \sum_{l=0}^{n-j} c_{j,n,l} \mathcal{L}_{j+l}(M_1)\mathcal{L}_{n-l}.$$

The integral of the Euler characteristic is recovered using the Gauss – Bonnet theorem $\chi = \mathcal{L}_0$. Versions of the KFF hold for spaces of constant curvature λ as well, with suitable modifications of the Lipschitz-Killing curvatures which replace $\mathcal{L}_{(\cdot)}$ with $\mathcal{L}^{\lambda}_{(\cdot)}$ [5].

The connection between Gaussian random fields and the KFF comes from general theory of orthogonal expansions of Gaussian processes [2, 3] which says that any centered unit variance process can be expressed as

$$f(t) = \sum_{i=1}^{\infty} \xi_i \psi_i(t) \quad (7)$$

for IID $N(0,1)$ random variables ξ_i and suitable functions $\psi_i, i \geq 1$. Truncating and normalizing after n terms yields a mapping $\Psi\colon M \to S(\mathbb{R}^n)$, the unit sphere in $\mathbb{R}^n$. Combining $\Psi(M)$ and a fundamental result in probability, Poincaré's limit [8] shows that the expected Euler characterstic (3) can be obtained as a limit of the KFF [3, 18].

This idea extends to random fields that are functions of IID centered, unit variance Gaussian random fields. For instance, consider a χ^2_2 random field

$$h(t) = f_1^2(t) + f_2^2(t)$$

where $f = (f_1, f_2)$ is a vector of two IID Gaussian random fields. The EC heuristic applies equally well to a χ^2_2 random field as a Gaussian random field (though the accuracy is more difficult to establish for non-Gaussian random fields). Note that

$$M \cap h^{-1}[u, +\infty) = M \cap f^{-1}(D_u)$$
$$D_u = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \geq u\}$$

and the expected Euler characterstic can be expressed as some functional of D_u.

Following the Poincaré limit argument through yields a remarkable form for this functional. Let $\text{Tube}(D,\rho)$ be the tube of radius ρ around $D \subset \mathbb{R}^n$.

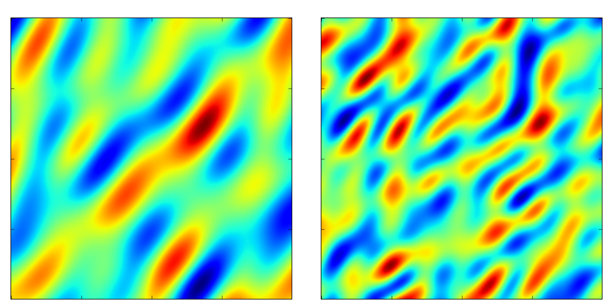
Theorem 2 (Gaussian Kinematic Formula in the plane, [3, 16])**.** Let $\gamma_{\mathbb{R}^n}$ be the distribution of $(Z_1, \ldots, Z_n) \sim N(0, I_{n\times n})$ and D be a C^2 domain in $\mathbb{R}^n$. Consider the following expansion in ρ

$$p_D(\rho) = \gamma_{\mathbb{R}^n}(\text{Tube}(D,\rho)) = \sum_{j=0}^{\infty} \frac{\rho^j}{j!} p_{j,D}.$$

Then,

$$\mathbb{E}\{\chi(M \cap f^{-1}D)\} = \sum_{j=0}^{\dim(M)} (2\pi)^{-j/2} \mathcal{L}_j(M) p_{j,D}.$$

The functionals $p_{j,\cdot}$ are finitely additive in D and can be expressed as limits of Lipschitz-Killing curvatures of warped products determined by D.



(a) (b)

Figure 4. (a) *A harmonisable stable process: note the strong direction associated to the first term in the moving average representation of the process.* (b) *A concatenated harmonisable process of order* 2, *which shows a random approximate lattice determined two dominant directions.*

Stable processes

The geometric properties of the excursions of Gaussian random fields, and functions of IID Gaussian random fields can be understood through the KFF and the GKF in Theorem 2. The relation between the supremum distribution (1) and the expected Euler characteristic (3) relies on the idea that it is very rare to have more than one critical values of f at high levels. In stable random fields [13], this argument breaks down and Theorem 1 fails [4]. In this case, the geometry of the excursion sets is driven largely by the shape of the kernel in the moving average representation of the process f. For instance, harmonisable processes are strongly directional, as depicted in Figure 4.

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Electrodes on the Brain? No Thank You!

by Tony Humphries (McGill University)

The recent thematic semester on Applied Dynamical Systems probed the latest uses of dynamical systems techniques in many application areas. From workshop to workshop, day to day, and talk to talk, it was a journey to many of the frontiers of science. In the description of the semester as a whole, there was not space to give more than a list of some of the topics discussed, and I wanted to give a sense of what was going on in the semester by looking at one area in more detail. There are many interesting subjects on which one could have focused. I picked deep brain stimulation and Parkinsonian tremor, which were covered in a theme session of the Mathematical Neuroscience workshop, mainly because the idea of sticking electrodes in peoples' heads makes my skin crawl... But that does not mean that there is not good mathematics there, as we will see.

On 30th January 2008, a piece of Canadian scientific research is making big news around the world. The English Independent newspaper even devoted its entire front page to the story, that Professor Lozano and his team at Toronto Western Hospital had managed to stimulate vivid recall of thirty year old memories in a patient by sticking electrodes into his brain and applying alternating current, and hailed this as the discovery of a potential way to reverse memory loss. The technique of applying AC current deep in the brain is known as deep brain stimulation (DBS), and its mathematics was studied during a special session on Parkinson's disease and DBS of the Mathematical Neuroscience workshop which took place as part of the CRM Thematic semester on Applied Dynamical Systems. Actually Professor Lozano's discovery was quite accidental, as he was probing for the point in the brain that controls appetite, in an attempt to reduce the appetite of the morbidly obese patient; the obesity treatment, unfortunately, failed.

This story shows not only that Canada remains at the forefront of neurological research, as it has since at least the time of Penfield, but also that neuroscience's understanding of the brain is still sufficiently limited that the outcomes of procedures and experiments can still bear little relation to expectations. In fact, DBS has its roots in Wilder Penfield's work on epilepsy in Montreal in the 1930s. In the Montreal procedure developed by Penfield, the skull is opened and the brain probed electrically while patients are kept under a local anaesthetic, which allows for monitoring of their responses. Once the region where the seizures originate is located, the nerve cells in that region of the brain are removed or destroyed. More than half of the patients treated with the procedure were cured of their seizures.

In Parkinson's disease, the characteristic 3-6 Hz muscle tremor can be completely debilitating, and in the workshop we saw video of one patient, who was completely incapable of opening a cupboard door, let alone making his own breakfast, without treatment. Before DBS was developed standard treatments for Parkinson's symptoms included various pharmocological or surgical interventions. Surgical treatment consisted of locating and destroying specific brain areas, such as the globus pallidus. Indeed, such pallidotomies were discovered by chance in 1952, when Irving Cooper destroyed one patient's globus pallidus by accidentally severing his anterior choroidal artery. He aborted the operation, ligated the artery, sewed up the patient, and hoped for the best. Here, presumably, "best" meant survival of the patient, so both patient and surgeon were very surprised when the patient came to, and his Parkinsonian tremor had disappeared. Pallidotomy became a standard treatment for Parkinson's symptoms until the development of drug treatments, which involve drugs that are converted to dopamine in the brain, to compensate for the deficiency of dopamine observed in Parkinson's disease patients. But, the effectiveness of drug treatments diminish over time, as patients develop tolerance to the drugs, and detrimental motor effects may result from increased dosages. So in the late 1980s, Alim Benabid ventured back inside patients' heads with electrodes, probing for the globus pallidus, much as Penfield probed for the source of epileptic seizures fifty years before. Benabid noticed that when he located the globus pallidus with the electrodes, the patients' tremors were reduced by the associated electrical stimulation. Even better results were obtained by introducing chronic stimulation with implanted electrodes, leading directly to DBS, which has since been used to treat over 40 thousand patients worldwide. Unlike previous treatments where various parts of the brain were permanently ablated, DBS is a reversible treatment as the current can be turned off. Indeed, electrodes implanted in the brain are connected to a control unit in the collarbone area, which allows for transdural tuning of stimulation parameters or turning off of stimulation. In the video mentioned earlier, when the patient's device was switched on, his tremor vanished, his mobility returned, and in true Canadian fashion, he was even seen shoveling snow.

This is all a great success story with just one small problem. DBS was developed as a surgical technique through observation and experiment. Despite its widespread use, there is no accepted explanation even amongst neurologists as to why or how it works; it is just known that it does. But the targets of DBS are networks of coupled neurons with their own intrinsic activity patterns. Thus, the impact of DBS can be modeled mathematically and analyzed using dynamical systems techniques. Jon Rubin, one of the organisers of the workshop, was part of one of the first teams to do that, and I asked him to describe some relevant mathematical issues further. This is what he had to say:

Initially, it was noted that the impact of DBS on motor symptoms was similar to that of surgical destruction, or ablation, of cells in the *target area. Thus, it was assumed that DBS introduced a functional ablation, meaning that it shut off the activity of target cells. This still appears to be a majority view among clinicians today; however, it leaves unanswered the question of how the removal of cells that are normally active helps improve motor performance. Moreover, several recent lines of evidence argue against functional ablation. In particular, Warren Grill and Cameron McIntyre, two of the speakers in the Mathematical Neuroscience session, showed in a biologically detailed neuronal model that, in a neighborhood of the stimulating electrode, DBS will silence each cell's soma while driving activity in its axon. The axon is the part of a neuron that sends signals to other cells, so this result suggests that DBS enhances output from its direct targets. This prediction is consistent with certain experimental findings that used artifact subtraction techniques to examine firing patterns in neurons of Parkinsonian monkeys during DBS.*

Howeevr, accepting the idea that DBS drives activity does not immediately yield a natural hypothesis about how it works. Indeed, DBS for Parkinson's disease (PD) typically targets cells that promote output from the basal ganglia, a primary site of dopamine action. This output is inhibitory, which means that it typically suppresses the activity of the cells to which it is sent. In PD, inhibitory basal ganglia outputs are enhanced relative to normal conditions. Thus, a paradox arises: if increased inhibition from the basal ganglia contributes to motor symptoms, then how could an additional increase, due to DBS, help alleviate these symptoms? To help resolve this paradox, a variety of authors, including most of the sessions' speakers, have proposed various theories focusing on the pattern of activity, rather than just the amount, across normal, Parkinsonian, and DBS conditions, and that's where dynamical systems come in.

My work, together with David Terman and more recently Cameron McIntyre, both of whom spoke in the session, and other collaborators, has used multiple timescale analysis to show how pathologically rhythmic outputs associated with PD can interfere with proper function of cells directly downstream from the basal ganglia, while the patterns of outputs seen under DBS restore this function. Alternatively, Warren Grill and his collaborators have characterized the entropy and information in the basal ganglia output signals under various conditions. Their findings have led them to propose that DBS induces an "informational lesion," replacing pathological levels of entropy with a low-information signal. A major force in analyzing DBS, and in proposing improvements to current DBS paradigms, has been the group of Peter Tass. This group focuses on the abnormally high degree of synchrony between cells in PD and has illustrated in a variety of coupled oscillator models how various forms of stimulation can lead to long-term desynchronization. Intriguingly, Tass's group has novel results on vanishing stimulation approaches to DBS, based on the idea that appropriate stimulation can alter communication

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Non-linear Integral Transforms: Fourier – Mukai and Nahm

by Benoit Charbonneau (McGill University and Duke University)

In August 2007, the CRM held the workshop *Non-linear Integral Transforms*: *Fourier – Mukai and Nahm* organized by Benoit Charbonneau (McGill/Duke), Jacques Hurtubise (McGill), Marcos Jardim (UNICAMP, Brazil), and Eyal Markman (UMass Amherst).

The transforms that were the subject of this workshop operate on moduli spaces, either of holomorphic objects or of gauge fields, and have been extensively developed over the past 20 years as privileged tools in the area.

The Nahm transform was initially introduced by Nahm in the early 80s to study magnetic monopoles. It developed over the years into a duality among instantons which are invariant under the action of a subgroup of translations of $\mathbb{R}^4$. On the other hand, the Fourier – Mukai transform was also introduced in the early 80s by Mukai as a duality among sheaves on abelian varieties. In the late 80s it was realized that both constructions are actually equivalent in certain circumstances. Another common feature is their role in mathematical physics, notably gauge theory and string theory.

The workshop gathered together a diverse group of people working on this fairly focused but current topic. We had a broad variety of participants: algebraic geometers, differential geometers and mathematical physicists. The topics of talks varied accordingly, firmly establishing the deep relevance of the ideas first uncovered by W. Nahm and S. Mukai 25 years ago.

The workshop was a great success. A large number of new ideas on Nahm transform, Fourier – Mukai transform, ADHM equations, translation invariant instantons, instantons on ALF spaces, and elliptic varieties have emerged in the various talks.

Most participants were speakers and most attended every talk. Many left Montréal ready to start working on a couple of new projects. It is our impression that some collaborations among the participants will emerge. There seems to be a need for a follow-up conference in a couple of years.

Thematic Semester on Applied Dynamical Systems

by Tony Humphries (McGill University)

The 2007 CRM thematic semester on Applied Dynamical Systems took place from June to December 2007. There were five workshops, two mini-courses, two advanced courses and two Aisenstadt Chairs at the CRM, and one workshop and a hurricane in Halifax!

Applied Dynamical Systems is a very broad field, and this semester focused on two themes. Firstly, the use of dynamical systems in applications, principally in physiology, and secondly the development of new numerical and dynamical systems tools needed in the study of problems arising in applications. However, in reality applications, analysis and numerical methods are all interconnected and aspects of all three permeated throughout the semester. Although a deep and beautiful theory of nonlinear dynamical systems has now been developed, systems arising in applications often fall outside the scope of this theory, because they contain generalizations to the dynamical systems paradigm, such as variable or state dependent delays or noise, or because they cannot be shown to satisfy conditions of the theory. Despite the difficulties, significant progress has been made in recent years in applying dynamical systems, particularly in the areas of mathematical biology and physiology. This has led to an increasing interest in new problems, such as those with non-constant and distributed delays, which has given rise to new analytical and numerical challenges.

The semester began with a mini-course on *Advanced Numerical Techniques in Applied Dynamical Systems* and a workshop on *Advanced Algorithms and Numerical Software for the Bifurcation Analysis of Dynamical Systems*, organized by E. Doedel (Concordia) and H. Osinga (Bristol), which took place from June 30 to July 7, 2007. The workshop was organized in daily themes, which were partial differential equations, structured dynamical systems, delay and functional differential equations, manifolds, and ordinary differential equations and multiple time scales. The last two days celebrated Sebius Doedel's 60th birthday, starting with a presentation by J. Guckenheimer (Cornell) on *Computing Multiple Timescale Dynamical Systems*, and culminating with a celebratory dinner and the surprise presentation to Sebius of a book on *Numerical Continuation Methods for Dynamical Systems* published especially for the workshop in order to celebrate his continuing influence in the field. Sebius' surprise doubled when he opened the book to discover that he had himself unknowingly supplied the first chapter. Herb Keller, who passed away on January 26, 2008, wrote the forward to the book, and so this meeting was sadly the last occasion for many of us to meet him.

A Workshop on *Mathematical Neuroscience*, held from September 16 to September 19, 2007, was organized by S. Coombes (Nottingham), A. Longtin (Ottawa), and J. Rubin (Pittsburgh). Mathematical neuroscience is an interdisciplinary field which aims to organize data from neural systems and model their behavior and function in both normal and pathological conditions. Through such investigations, fundamental knowledge is gained about the principles of neural function at various spatial and temporal scales, new experiments are proposed and new therapies are derived for correcting neurological disorders. The workshop highlighted recent developments in applied mathematics, especially in nonlinear differential equations and stochastic dynamics, that have been driven by problems in neuroscience. The event also included two theme sessions giving an overview of mathematical modeling issues in two focus areas: information processing in the auditory system, and Parkinson's disease and deep brain stimulation (DBS). The session on Parkinson's disease and DBS is described in a separate article.

In the auditory system session, I. Bruce (McMaster) presented methods to analyze the influence of noise on spike initiation and how such threshold fluctuations can be obtained in electrical stimulation from cochlear implants. J. Rinzel (NYU), in one of his Aisenstadt presentations, used concepts from dynamical systems and coding theory to explain phasic firing, precise phase locking and extremely timing-sensitive coincidence detection involved in sound localization. G. Chechik (Stanford) presented a study of redundancy reduction and information transfer about the spectro-temporal content of auditory stimuli, showing how redundancy decreases as one moves to higher brain centers. M. Elhilali (Maryland) discussed a computational framework for segregating sounds into meaningful streams based on temporal coherence. L. van Hemmen (Munich) showed how inputs from one sense can act as a learning supervisor for another sense, in the context of spike-time dependent plasticity.

Other topics covered in the workshop included single cell properties and their influence on network activity, patterns of activity such as propagating waves and synchronous firing in neural excitable media, synchronization transitions and stochastic dynamics.

Continuing the biological theme, the workshop on *Deconstructing Biochemical Networks*, held on September 24–28, 2007, and organized P.S. Swain (McGill), B.P. Ingalls (Waterloo) and M.C. Mackey (McGill), was preceded by a mini-course on *Quantitative Biology* on September 22-23, 2007.

The mini-course, attended by more than thirty graduate students, postdoctoral fellows and other researchers, was an introduction to some of the fundamental concepts of systems biology. The first session, given by the Aisenstadt chair John Tyson, was an introduction to the use of ordinary differential equations modeling and accompanying dynamical systems analysis. The following sessions took up a number of timely topics in the analysis of cell biological systems: stochastic modeling, methods for data fitting, synthetic biology and control theory.

The workshop attracted a large number of high profile speakers and many of the talks addressed how important biological problems could only be solved with a combination of mathematics and experiments. Two major mathematical themes emerged: the need to model stochastic dynamics and the need to identify simple, underlying dynamical systems driving the dynamics of large networks of interacting genes and proteins. The origin of oscillations with robust periods in biochemical networks and how cells process information in their stochastic intracellular environments were topics that many attendees raised and were thoroughly discussed. Speakers included experimental biologists, applied mathematicians, computer scientists, control engineers, and biophysicists. The atmosphere was very collaborative with many questions, and the meeting culminated in a busy poster session, which allowed nearly all the students and postdoctoral fellows to present their work.

Many of the themes covered in these two workshops, will be revisited in the SIAM Life Sciences conference which will be held in Montréal in August 2008, co-chaired by J. Rubin (Pittsburgh) and S. Cox (Rice), who both participated in the Mathematical Neuroscience workshop.

The Joint AARMS-CRM Workshop on *Recent Advances in Functional and Delay Differential Equations* took place in Halifax on November 1-5 2007. It was organised by H. Brunner (Memorial), A.R. Humphries (McGill) and D.E. Pelinovsky (McMaster), with local organizers P. Keast (Dalhousie) and P. Muir (Saint Mary's). A fairly mature theory of constant delay equations as infinite dimensional dynamical systems has been developed. However, models in physical and biological applications are increasingly encompassing features which do not fit this theory, often having non-constant and state-dependent delays, or advanced arguments, and Volterra functional (integral and integro-differential) equations are also applied with increasing frequency. The theory of such problems is still far from complete, though significant progress is being made. A large gap also exists in the numerical analysis and computational solution of such functional equations. This workshop brought together researchers and students from applied, numerical and theoretical viewpoints, and gave a wide perspective on recent results, current research and open problems, in these overlapping fields.

One theme that ran through the workshop was the use of delay equations in applications, particularly biological applications. Here it would be natural for the delays to be state dependent, but most models treat delays as constant, mainly because of the lack of techniques and theory for the state dependent problems. It is clear that provided mathematicians can supply the theory and methods to solve the problems, biological applications will provide a plentiful source of new state-dependent delay problems.

From a theoretical viewpoint, existence results for periodic solutions, in particular slowly oscillating periodic solutions (with period larger than twice the delay), were presented under a variety of situations, including negative and positive feedback problems. Numerical methods are also under development for state-dependent problems, where the problem of solution termination for neutral problems has come under investigation. Methods were shown to continue solutions beyond termination using either a Filipov-like set-valued extension of the differential equation, or by regularization. The relevance of these methods was shown in an example where a solution passed through several terminations before converging to an attractive periodic orbit in a region of phase space without terminations. Another area of recent complementary developments in theory and numerics concerns stability of fixed points, where efficient methods for the numerical computation of characteristic values were presented alongside the theory being developed to link nonlinear and linear stability of fixed points for state-dependent problems (which is a much harder problem than in constant delay equations).

The talks on Volterra integral, integro-differential and more general functional equations exposed the state of the art of the numerical treatment of such equations. In particular, a general approach based on the abstract representation of the numerical solution to Volterra equations (of parabolic or non-parabolic type) in terms of the analytical solution leads to a comprehensive numerical stability analysis. In the case of Volterra integral or integro-differential equations with weakly singular kernels, it appears advantageous to subject them to a "smoothing transformation"; the resulting improved regularity of the solutions leads to more efficient numerical methods. But, many numerical issues remain to be solved, including analysis of efficient methods for Volterra equations with highly oscillatory kernels, and the design of reliable numerical codes for Volterra-type functional differential and integral equations including state-dependent problems.

Also of considerable interest, were nonlinear traveling wave problems on lattices, arising from materials science, atomic physics and nonlinear optics. These lead to dissipative or Hamiltonian advanced-retarded equations on the entire axis with homoclinic, heteroclinic or periodic solutions. In the Hamiltonian case, recent progress has been made using center manifold reductions and normal form transformations. Many other problems and results were presented, as partial differential equations with delays, where there is potential for significant future attention, and the Wheeler-Feynman problem of classical electrodynamics.

The meeting was jointly organized by CRM and AARMS and took place in Halifax, initially at Dalhousie University. As well as a strong turnout of the Canadian delay equations community, many participants traveled from Europe, and some from as far afield as Brazil, New Zealand and Estonia. It was great success at getting researchers from different fields and countries to interact. One unwelcome guest was Hurricane Noel, whose remnants swept across Halifax on Saturday night, soaking everyone walking back to the hotel after the seminars. Worse still, Sunday morning we discovered that Dalhousie was in one of the powerless parts of the city, and we had lost our venue. So, we all owe a big vote of thanks to Pat Keast, who saved the conference by arranging an alternative venue at the Lord Nelson Hotel on Sunday morning, at less than an hour notice. In case you were wondering, the CRM does not have Hurricane insurance! In all, it was a very memorable workshop.

A workshop on *Dynamical Systems and Continuum Physics* held at the CRM November on 14-16, 2007, and organized by L. Tuckerman (PMMH-ESPCI, France), focused on recent research in laminar and turbulent hydrodynamics and in some more discrete systems, notably granular media and foams. In laminar hydrodynamics, transitions in rotating, sheared, and heated fluids provided motivation for the development of bifurcation theory during much of the 20th century. Attention has now shifted to more complicated scenarios involving tori and heteroclinic orbits, and to open flows such as wakes and shear layers. Sessions were held on hydrodynamics, Faraday patterns, transition to chaos, granular media, and foam.

In the session on rotating flows, J. Lopez (Arizona State) proposed a method to control vortex breakdown, which creates a stagnation point rotating flow, and which can have serious consequences in aerodynamics; F. Marques (Politecnica de Catalunya) spoke about complex dynamical processes (bursting, heteroclinic and homoclinic orbits) in Taylor–Couette flow between differentially rotating concentric cylinders, and P. Le Gal (IRPHE) showed how several different instabilities could be viewed as manifestations of unifying general principles.

In the session on convective pattern formation, E. Knobloch (Berkeley) presented his discovery of convectons, which are localized regions of convection which grow by adding rolls via homoclinic "snaking"; S. Morris (Toronto) spoke on electroconvection in smectic liquid crystals, which produces two-dimensional vortices, and L. Tuckerman (PMMH-ESPCI, France) described an analogy between the linear and nonlinear problems of binary fluid convection, whereby both the growth rates and the energy are roots of quadratic equations.

The Faraday problem, concerning patterns formed on the surface of a vertically vibrated fluid layer, had a Renaissance in the early 1990s when it was discovered that temporal oscillations combining multiple frequencies lead to spatial quasipatterns. C. Huepe (Northwestern) analyzed the temporal forcing function with the WKB approximation, and applied this analysis to the "inverse Faraday problem" of finding a forcing that produces a given spatial wave pattern. J. Conway (Northwestern) discussed superlattice patterns and their stabilization, and J. Viñals (McGill) discussed the dynamics of diblock copolymers.

The Navier–Stokes equations continue to govern turbulent hydrodynamics; the challenge is to explain the transition to chaotic behavior, i.e., turbulence, displayed by these well-known deterministic equations, especially in the absence of linear instability. Patterns involving multiple turbulent structures or the coexistence of turbulent with laminar regions provide interesting new puzzles. One session was devoted to phenomena occurring near transition to turbulence in wall-bounded flows, and included talks by B. Eckhardt (Marburg) and D. Barkley (Warwick)

Granular media and foams, on the cutting edge of current continuum physics, are not as well understood as hydrodynamics. In the foams session, P. Marmottant (CNRS/Grenoble 1) described attempts to formulate continuum equations, combining elastic, plastic, and viscous properties; M. Dennin (Irvine) discussed recent experiments on "rafts" of bubbles floating on water, which shed light on solid, liquid and plastic properties of foams, and J. de Bruyn (Western Ontario) presented experiments to measure drag in foams. In the granular media session, S. Morris (Toronto) discussed the well known but poorly understood phenomenon of segregation, whereby a mixture of grains of different sizes separates when shaken, stirred or turned; O. Dauchot (CEA/Saclay) investigated spatial and temporal correlations in granular media via experiments on clusters, relaxation, aging, jamming and flow, and J. de Bruyn (Western Ontario) described experimental and theoretical work on impact craters in sand. C. Radin (UT Austin) discussed the relationship between sphere packings and granular media in two talks, the second a colloquium talk which closed the meeting, on non-optimally dense packings of spheres and their applications to granular media.

The semester concluded with a workshop on *Chaos and Ergodicity of Realistic Hamiltonian Systems*, held on December 11-14, 2007, and organized by H. Broer (Groningen) and P. Tupper (McGill). The workshop brought together experts in dynamical systems from mathematics, physics, and computer science to present their work on Hamiltonian systems. Two main types of models were considered: atomistic dynamics and celestial mechanics. The former ranged from detailed molecular dynamics simulations of biochemical systems to model systems intended to probe the foundations of statistical mechanics. The latter stretched in scope from the dynamics of interacting galaxies down to the subtle mechanisms that form Saturn's rings.

Some consistent themes emerged. Firstly, the concept of ergodicity usually considered in dynamical systems (such as that shown for geodesic flows on manifolds of negative curvature and dispersive billiards) does not appear to be appropriate for systems arising in many applications. Many investigations presented at the workshop revealed that the physically important statistical behavior occurs on a long, but finite, time scale. If the truly infinite time limit of the system were considered, the system would have very different behavior. Beside many systems where computational experiments revealed this to be true, a theoretical example was presented in the talk of M. Shub (Toronto) on *Stable Ergodicity*. One result of this theory

(*continued on page* 22)

Activités 2008 Activities

Thematic Programs

Thematic Semester on Dynamical Systems and Evolution Equations

January – June 2008

www.crm.math.ca/Evolution2008

Young Mathematicians' Conference
January 18 – 19, 2008

Workshop on Initial Conditions
January 24 – 25, 2008

Workshop on Spectrum and Dynamics
April 7 – 11, 2008

Workshop on Geometric Evolution Equations
April 16 – 27, 2008

André-Aisenstadt Chair
Gerhard Huisken (Max-Planck-Institut für Gravitationsphysik)
April 2008

André-Aisenstadt Chair
Jean-Christophe Yoccoz (Collège de France)
May 5 – 8, 2008

Workshop on Singularities, Hamiltonian and Gradient Flows
May 12 – 16, 2008

Workshop on Floer Theory and Symplectic Dynamics
May 19 – 24, 2008

Thematic Year on Probabilistic Methods in Mathematical Physics

June 2008 – June 2009

www.crm.math.ca/Mathphys2008

Workshop on Mathematical Aspects of Quantum Chaos
June 2 – 7, 2008

Workshop on Integrable Quantum Systems and Solvable Statistical Mechanical Models
June 30 – July 5, 2008

Workshop on Stochastic Loewner Evolution and Scaling Limits
August 4 – 9, 2008

André-Aisenstadt Chair
Wendelin Werner (Paris-Sud, Orsay)
August 4 – 16, 2008

Workshop on Laplacian Growth and Related Topics
August 18 – 23, 2008

Workshop on Random Matrices, Related Topics and Applications
August 25 – 30, 2008

André-Aisenstadt Chair
Craig Tracy (UC Davis)
August 26 – 31, 2008

Workshop on Random Tilings, Random Partitions and Stochastic Growth Processes
September 1 – 6, 2008

André-Aisenstadt Chair
Andrei Okounkov (Princeton)
September 1 – 16, 2008

Workshop on Quantum Many-Body Systems; Bose — Einstein Condensation
September 29 – October 4, 2008

Workshop on Random Functions, Random Surfaces and Interfaces
January 4 – 9, 2009

Workshop on Interacting Stochastic Particle Systems
May 18 – 23, 2009

Workshop on Disordered Systems: Spin Glasses
June 8 – 13, 2009

CRM – PIMS Joint Thematic Program: Challenges and Perspectives in Probability
October 2008 – September 2009

www.pims.math.ca

André-Aisenstadt Chair
Svante Janson (Uppsaala)
October 13 – 25, 2008

Workshop on Combinatorics, Randomization, Algorithms and Probability
May 4 – 8, 2009, CRM

Workshop on New Directions in Random Spatial Processes
May 11 – 15, 2009, CRM

Workshop on Random Walks in Random Environments
June 1 – 5, 2009, PIMS-UBC

Summer School "Renormalisation Group and Related Topics" & "Stochastic Population Systems"
June 8 – July 3, 2009, PIMS-UBC

PIMS Distinguished Chair
Don Dawson (Carleton and McGill)
June 8 – July 3, 2009, PIMS-UBC

Workshop on the Renormalization Group and Statistical Mechanics
July 6 – 10, 2009, PIMS-UBC

Workshop on Mathematical Challenges from Molecular Biology and Genetics
September 6 – 10, 2009, BIRS

General Scientific Activities

2e Conférence francophone sur les architectures logicielles
14e Colloque international sur les langages et modèles à objets
3 au 7 mars 2008, Université de Montréal

Grande conférence CRM
Tadashi Tokieda (Cambridge)
« Dimension 2 $\frac{1}{2}$: Science à partir d'une feuille de papier »
18 mars 2008

The 5th Montréal Science Computing Days
April 30 – May 2, 2008

First CRM – INRIA – MITACS Meeting
May 5 – 9, 2008

Theory CANADA 4 Conference
June 4 – 7, 2008

A Celebration of Raoul Bott's Legacy in Mathematics
June 9 – 13, 2008

Colloquium of Non-Commutative Algebra
June 9 – 14, 2008, Université de Sherbrooke

The Eight Canadian Summer School on Quantum Information
June 9 – 16, 2008, Université de Montréal

SMS 2008 Summer School
Symmetries and Integrability of Difference Equations
June 9 – 21, 2008

8th International Conference on Symmetries and Integrability of Difference Equations (SIDE 8)
June 22 – 28, 2008, Sainte-Adèle (Québec)

Eight International Conference on Monte Carlo and Quasi-Monte Carlo Methods in Scientific Computing
July 6 – 11, 2008

Conference on Model Subspaces
September 15 – 21, 2008

2008 CRM – Fields – PIMS Prize

(*continued from page* 8)

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Thematic Semester

(*continued from page* 19)

is that generic high-dimensional systems may be ergodic as long as they are coupled, no matter how weakly, to a sufficiently strongly hyperbolic system. However, the time scale on which this ergodicity would be observed is immense and would likely be beyond the time scales of theoretical interest for many applications.

Paralleling this theme was the question of, given a high-dimensional Hamiltonian system, what low-dimensional functions of the system does one observe. Ergodicity is framed in terms of time averages of virtually all observables of a system. Many of the speakers showed that the statistical behavior observed depends to a large extent on which functions are considered. An interesting example of this was the talk of A. Giorgilli (Milano) on the Fermi – Pasta – Ulam problem where measures of mixing in the equilibrium state had a delicate dependence on which observables were considered.

Perhaps the most realistic Hamiltonian systems discussed in the workshop were in the talk of W. Hayes (Irvine) on chaos in the solar system. A long-standing question in celestial mechanics is whether the solar system is chaotic. Two well-respected computational groups have attacked this problem and came up with different answers. Hayes' detailed study of the system using high-accuracy numerical methods has resolved the apparent contradiction. It appears that the equations modeling the solar system have trajectories with both zero (non-chaotic) and non-zero (chaotic) Lyapunov exponents. Surprisingly, the current state of the solar system is not known accurately enough to determine the type of trajectory. The two different groups started with slightly different initial conditions, both consistent with the data, and arrived at different conclusions.

Two advanced graduate courses were also associated with the theme semester: *A Practical Introduction to SDEs*, taught at McGill University by P. Tupper, and *Numerical Analysis of Nonlinear Equations* taught at Concordia University by E. Doedel.

Electrodes on the Brain?

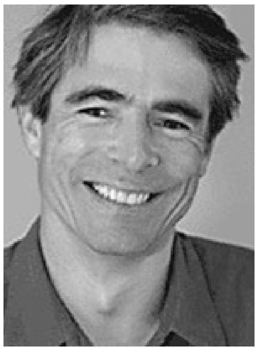
(*continued from page* 16)

between neurons in such a way that the synchronous state destabilizes, eliminating the need for subsequent stimulation. Tass presented some modeling, analysis, and even preliminary clinical results on these ideas in the workshop. The final workshop speaker, David Hansel, did not touch on DBS but did offer a theory, based on analysis of coupled firing rate models, on how disrupted competition between different large-scale pathways across multiple brain areas could underlie certain motor symptoms of PD. These ideas could either complement or stand as an alternative to the induction of symptoms through locally emergent rhythmicity, but their implications for DBS remain to be explored.

Jon Rubin will return to Montréal in 2008 to co-chair the SIAM Life Sciences Conference with Steve Cox (Rice), which will take place at the Hyatt Regency, Montréal from 4th to 7th August 2008.

Au revoir

François Lalonde, directeur du CRM



Je quitte la direction du CRM le 31 mai 2008 et le Canada pour un certain temps. Dans cette expérience de quatre ans au CRM et de quinze ans dans d'autres initiatives, ce qui me vient immédiatement à l'esprit est la joie que j'ai éprouvée à travailler avec les mathématiciens du monde entier, mais aussi avec les physiciens, les informaticiens (classiques, probabilistes et quantiques), certains chimistes, quelques statisticiens et des ingénieurs, des biologistes, et une belle équipe transatlantique en imagerie cérébrale et de la moelle épinière. J'ai tenté de les comprendre. Nous étions animés d'un enthousiasme que l'on ne trouve peut-être pas si souvent, même dans le Nouveau Monde. Cela a mené quelques fois, lorsque nous étions chanceux, à de belles découvertes.

Je suis profondément reconnaissant au comité scientifique consultatif du CRM : à Jerry Bona qui est devenu par conviction un ami du CRM dans le monde, à Jean-Louis Colliot-Thélène, à Jean-Pierre Bourguignon, à Steve Zelditch, à Mark Haiman, à Mitchel Luskin, à Richard Lockhart, à Carl Pomerance, à Walter Craig, à Jim Berger, à Alice Guionnet, à Mark Goresky, à Tom Salisbury, à Catherine Sulem, et à Jean-Christophe Yoccoz. Ils sont tous venus aux réunions, peu importe la distance à parcourir. . .

Alors que le CRM a presque doublé le nombre et l'ampleur de ses programmes depuis 2002, le nombre d'employés du CRM a chuté de 14 à 11 – c'est dire combien je dois à ce personnel lucide et dévoué. C'est dire combien ils travaillent et combien ils y croient.

Il y a maintenant dix laboratoires associés au CRM, répartis sur 12 campus universitaires dans la moitié est du Canada. Il y a aussi 1 500 visiteurs chaque année dans nos programmes thématiques. Il y a 250 scientifiques impliqués aujourd'hui dans l'organisation des onze programmes du CRM (la programmation thématique et les laboratoires ne sont que deux d'entre eux, ce sont les plus importants, mais il y en a plusieurs autres). Plus de la moitié de ces scientifiques sont hors Canada. C'est pour eux, canadiens et étrangers, que j'ai travaillé. On a quelques fois tendance, au Canada, à concevoir les directeurs d'instituts comme des généraux à la tête d'armées de scientifiques mathématiciens. Je ne me suis pas reconnu dans cette image ; c'est moi qui était au service de chacun. Les mathématiques naissent le plus souvent dans l'intimité la plus farouche. Elles s'imposent à nous quand notre être tient tout entier dans un lieu étroit. Ce sont ces cours ombragées, modestes, quelques fois têtues qui sont les seuls palaces du monde. Et combien il est tentant de croire que le monde n'est vaste que parce qu'il contient tant de ces petites tanières.

Le CRM est vigoureusement soutenu par ses universités partenaires au Canada, par le CRSNG (Canada), le FQRNT (Québec), la NSF, le programme *Science for Peace* (Bruxelles), et plusieurs autres organismes. Le CRM vient par ailleurs de signer des accords avec l'Union européenne (programme Erasmus), l'INRIA, l'INSERM , le CNRS, plusieurs organismes aux États-Unis, l'Asie et soutient les scientifiques de La Havane. Notre accord avec le Tata Institute est à ce jour le principal résultat de la mission ministérielle du Québec en Inde.

Finalement, je n'aurais pas accepté ce poste si je n'avais pas eu le bonheur de travailler avec Barbara Keyfitz (Fields), Ivar Ekeland (PIMS), Octav Cornea et Alexandra Haedrich (Institut des sciences mathématiques, l'une des premières écoles doctorales unifiées dans le monde occidental), ainsi qu'avec ceux et celles qui les ont accompagnés. Je suis reconnaissant aux directeurs adjoints du CRM, surtout Odile Marcotte (UQÀM) qui a pris en charge le réseau industriel du CRM, Chantal David (Concordia), Jean LeTourneux (Montréal), Andrew Granville (Montréal), et au directeur qui m'a précédé et qui a beaucoup fait pour le CRM, Jacques Hurtubise (McGill), ainsi qu'à des centaines d'autres personnes de toutes les régions du Canada et du monde, que je ne peux pas toutes nommées ici. Je laisse le CRM au moment où les quatre prochains programmes thématiques sont établis – ils nous mèneront jusqu'en 2010. Le CRM aura les ressources pour les soutenir. Au moment d'écrire ces lignes, nous ne savons pas qui prendra la direction du CRM le 1er juin 2008.

François Lalonde, le 4 mars 2008

Farewell

François Lalonde, Director of CRM

On May 31, 2008, I shall be leaving my position as director of the CRM, as well as Canada for a period of time. Thinking back on my four years at the CRM and fifteen years working on other initiatives, what first comes to mind is what a great pleasure it has been to work with mathematicians from around the world, as well as physicists, (classical, probabilistic and quantum) computer scientists, certain chemists, several statisticians and engineers, biologists, and a splendid transatlantic team of scientists working on brain and spinal cord imaging. I tried to understand them. We were driven by a shared enthusiasm that is not so common, even in the New World. Sometimes, when we were lucky, this lead to great discoveries.

I am deeply grateful to the CRM Scientific Advisory Panel: to Jerry Bona, who by conviction has become a great friend to the CRM, to Jean-Louis Colliot-Thélène, Jean-Pierre Bourguignon, Steve Zelditch, Mark Haiman, Mitchel Luskin, Richard Lockhart, Carl Pomerance, Walter Craig, Jim Berger, Alice Guionnet, Mark Goresky, Tom Salisbury, Catherine Sulem, and Jean-Christophe Yoccoz. They all came to our meetings, despite the distances. . .

While the CRM has almost doubled the quantity and scale of its programmes since 2002, the number of employees has dropped from 14 to 11 — this is an eloquent testimony to how much I owe this lucid and devoted staff, how hard they work, and how much they believe in what they do.

There are now ten laboratories associated with the CRM, spread over twelve campuses in Eastern Canada. There are also 1500 visitors each year for our thematic programmes. About 250 scientists are involved in organizing the CRM's eleven programmes (the thematic programming and the laboratories are the two principal programmes, but there are several others). More than half of these scientists are from abroad. It is for these scientists, Canadian and foreign, that I worked. There is sometimes a tendency in Canada to consider the Institute directors as generals of an army of mathematical scientists. I never recognized myself in this image; rather, I was at the service of each of these scientists. Mathematics is almost always born in the fiercest intimacy. It imposes itself on us when we are completely confined in a small space. These shadowy, modest, sometimes stubborn spaces, are the world's sole palaces. It is most tempting to believe that the world is vast only because it contains so many of these small alcoves.

The CRM is vigorously supported by its partner universities, by NSERC (Canada), the FQRNT (Québec), the NSF (Washington), NATO civil program Science for Peace (Brussels), and several other institutions. The CRM has signed agreements with Erasmus programme (European Union), INRIA, INSERM, the CNRS, several organizations in the USA and Asia, and it supports scientists in Havana. Our agreement with the Tata Institute is the main outcome of the Québec Ministerial Mission to India.

Finally, I would never have accepted this position had I not had the pleasure of working with Barbara Keyfitz (Fields), Ivar Ekeland (PIMS), Octav Cornea and Alexandra Haedrich (Institut des sciences mathématiques, one of the first unified doctoral schools in the Western world), as well as those who work with them. I am grateful to the CRM deputy directors, especially Odile Marcotte (UQÀM) who took charge of the CRM industrial network, Chantal David (Concordia), Jean LeTourneux (Montréal), Andrew Granville (Montréal), and to my predecessor, Jacques Hurtubise (McGill), who did so much for the CRM, and to hundreds of other people from throughout Canada and the world, whom I cannot all name here. As I leave the CRM, the next four thematic programmes are organized — this will bring us to 2010. The CRM has the resources to support them. At the time of writing these lines, we do not know who shall take on the direction of the CRM in June 2008.

François Lalonde, March 4, 2008

Le Bulletin du CRM

Volume 14, Nº 1
Printemps 2008

Le Bulletin du CRM est une lettre d'information à contenu scientifique, faisant le point sur les actualités du Centre de recherches mathématiques.

ISSN 1492-7659

Le Centre de recherches mathématiques (CRM) a vu le jour en 1969. Présentement dirigé par le professeur François Lalonde, il a pour objectif de servir de centre national pour la recherche fondamentale en mathématiques et dans leurs applications. Le personnel scientifique du CRM regroupe plus d'une centaine de membres réguliers et de boursiers postdoctoraux. De plus, le CRM accueille chaque année entre mille et mille cinq cents checheurs du monde entier.

Le CRM coordonne des cours gradués et joue un rôle prépondérant en collaboration avec l'ISM dans la formation de jeunes chercheurs. On retrouve partout dans le monde de nombreux chercheurs ayant eu l'occasion de perfectionner leur formation en recherche au CRM. Le Centre est un lieu privilégié de rencontres où tous les membres bénéficient de nombreux échanges et collaborations scientifiques.

Le CRM tient à remercier ses divers partenaires pour leur appui financier dans notre mission : le Conseil de recherches en sciences naturelles et en génie du Canada, le Fonds québécois de la recherche sur la nature et les technologies, la National Science Foundation (États-Unis), le Clay Mathematics Institute (États-Unis), l'Université de Montréal, l'Université du Québec à Montréal, l'Université McGill, l'Université Concordia, l'Université Laval, l'Université d'Ottawa, l'Université de Sherbrooke, ainsi que les fonds de dotation André-Aisenstadt et Serge-Bissonnette.

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