CENTRE DE RECHERCHES MATHÉMATIQUES

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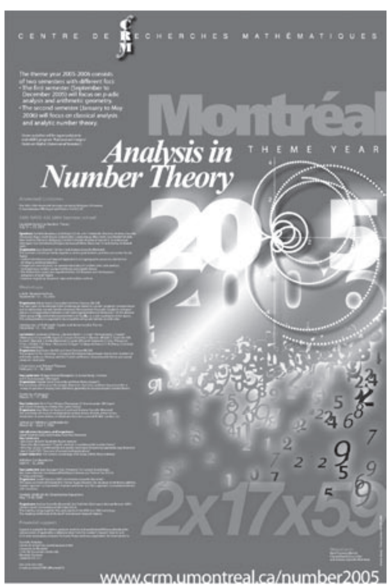
Analysis in Number Theory

by Chantal David (Concordia University)

The 2005-2006 Theme Year at Centre de recherches mathématiques (CRM) is entitled "Analysis in Number Theory". The program organisers are Henri Darmon (McGill), Chantal David (Concordia) and Andrew Granville (Montréal). The theme year will consist of two semesters with different foci, both exploring the fruitful interactions between analysis and number theory. In many areas of number theory, one is led to the study and understanding of analytic objects, such as the ubiquitous L-functions, which appear in various forms in different areas of number theory and arithmetic geometry.



The first semester will focus on p-adic analysis and arithmetic geometry, and the second semester on classical analysis and analytic number theory. In both themes, several workshops, schools and concentration periods will be dedicated to the new developments that have emerged from the interplay between the two fields. In addition to the participants of the six week-long workshops and the two schools held during the theme year, more than forty long-term visitors will be in Montréal for periods varying from two weeks to six months including many of the leading researchers in their fields. The 2005-2006 André-Aisenstadt chairs are Manjul Bhargava (Princeton), K. Soundararajan (Michigan) and Terry Tao (UCLA), who will be visiting the CRM for two months, six months and two weeks respectively. Some of the activities of the theme year are organised jointly with the program "Rational and Integral Points on Higher-Dimensional Varieties" held at the Mathematical Sciences Research Institute (MSRI) in Winter 2006.

The theme year began with the NATO Summer School on Equidistribution in Number Theory (July 11-22, 2005). This two-week school, organised by Zeev Rudnick (Tel Aviv) and Andrew Granville (Montréal), was primarily targeted at senior graduate students, postdoctoral fellows and junior faculty. The topics of the school included distribution of integers and applications, integral and rational points on varieties, equidistribution of complex multiplication points and Hecke points, points of small height, and quantum ergodicity.

The school was attended by more than 100 participants from around the world, and was a big success, due in part to the constant efforts made by the lecturers to explain the latest developments in the subject at a level accessible to the audience. The organisers are pleased to report that the attendance was as large on the last day than on the first one, a tribute to the efforts made by the lecturers!

A concentration period on p-adic representations is held in the Fall, and several distinguished long-term researchers are visiting the CRM from mid-August to mid-November. The concentration period began with seminars

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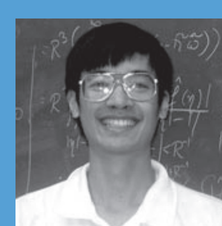
The 2005 - 2006 André Aisenstadt Chairs

Manjul Bhargava (Princeton) who has just received the Blumenthal Prize, will give a series of lectures from March 15 to May 15 2006.

K. Soundararajan (Michigan) will give a series of lectures during Winter 2006.



Terence Tao (UCLA) will give a series of lectures in April 2006.

The 2004-2005 André-Aisenstadt Prize

by Ravi Vakil (Stanford University)

The 2004-2005 André-Aisenstadt Prize was awarded to Ravi Vakil (Stanford University) on April 29, 2005. After completing his B.Sc. and M.Sc. at the University of Toronto (1992), Dr. Vakil obtained his Ph.D. from Harvard University (1997) under the supervision of Joe Harris. He then spent a year as a post-doctoral fellow at Princeton University and three years at MIT as a Moore Instructor, before becoming an Assistant Professor at Stanford in 2001. Dr. Vakil works in algebraic geometry, investigating the enumerative geometry of projective algebraic curves. In his most spectacular work, he studied degenerations in a Grassmannian, to solve several old problems in Schubert calculus. Several prizes and honors recognized his exceptional achievements, including a NSF Career Fellowship, a Sloan Research Fellowship, a Centennial Fellowship and a G. de B. Robinson Prize. Here is his research description.

We consider the question: "How bad can the deformation space of an object be?" (Alternatively: "What singularities can appear on a moduli space?") The answer seems to be: "Unless there is some a priori reason otherwise, the deformation space can be arbitrarily ugly." Hence many of the most important moduli spaces in algebraic geometry are arbitrarily singular. More precisely, up to smooth parameters, every singularity that can be described by equations with integer coefficients appears on the moduli spaces of: smooth surfaces (or higher-dimensional varieties); smooth curves in projective space (and the space of stable maps); plane curves with nodes and cusps; stable sheaves; isolated singularities; and more. The objects themselves are not pathological, and are in fact as nice as can be. This justifies a philosophy of Mumford. The complex-minded listener may work in the holomorphic category. The arithmetic listener may think in mixed or positive characteristic. Much of my research to date has involved the study of various moduli spaces, both as a means to other ends, and as an end in itself. My methods are algebro-geometric, but the motivations and ideas also come from other fields, including combinatorics, string theory, representation theory, symplectic geometry, and topology.

If you are interested in studying a certain kind of object, say Riemann surfaces of genus g, you will inevitably be led to the notion of a moduli space of such objects. A moduli space of a some type of object is, essentially, a parameter space of such objects. A prototypical example is projective space $\mathbb{P}^n$, parameterizing one dimensional sub-vector-spaces of an $(n+1)$-dimensional space K^{n+1}. (Here let K be your favorite field, for example $\mathbb{R}$ or $\mathbb{C}$. I prefer to be agnostic about my field, or even to "work over the integers", whatever that means.) The one-dimensional subspaces of K^{n+1} correspond to the non-zero vectors in K^{n+1} , modulo multiplications by non-zero scalars:

$$\mathbb{P}^n = \{[x_0; x_1; \ldots; x_n]\} / K^*.$$

At this point, we have just described projective space as a *set*. But if $K = \mathbb{R}$, then we intuitively have a notion that one-dimensional subspaces can be "near" each other, which is to say that we are expecting that $\mathbb{P}^n$ is in fact a topological space. Better yet, by covering projective space with nice open sets, we can describe it as a *manifold* (or even as an *algebraic variety*). Indeed, this description is often given (sometimes implicitly) in the first class in which projective space is introduced.

However, there is a subtle question here: who is to say that the topological (or manifold) structure is the "right" structure to impose on this set? Is it intrinsic and "God-given'', or arbitrarily imposed by man? The answer to this question (that there *is* an inherent "correct" structure) is subtle, and involves the correct definition of the notion of moduli space (what some call a "fine moduli space"). The Grothendieck school realized the correct definition in the 1960's. It is short to state, involving the notion of how the objects vary in families, but it is conceptually difficult to come to terms with, so I will not give a precise description here. Suffice it to say that if you have a certain kind of object you are interested in studying, then you may hope that there is a moduli space of such objects, and if there is a moduli space, then there is only one.

As an example, consider the *Grassmannian*, of k-dimensional subspaces of n-dimensional space. (The $k = 1$ case corresponds to projective space.) There is indeed a set of such objects, but in fact it has a much better structure: if we are working over the real numbers, there is a moduli space, which is a compact real manifold of dimension $k(n-k)$. (The same statement is true over the complex numbers, or indeed over any field, if the right definitions are made.) The Grassmannian is in some strong sense the geometric object behind linear algebra.

In general, if you have well-behaved objects, a moduli space will often exist. For example, if you have "nice" holomorphic objects, there is often a holomorphic moduli space. Even when there is no moduli space in your category of choice, all is not lost. There are two patches: weaken your definition of "moduli space" (using the notion of "coarse moduli space"), or generalize your definition of "space". In many respects the second road is the better one. As an example, there isn't quite a moduli space of Riemann surfaces in the complex-analytic category. But if you extend your notion of "space" a little bit, you can indeed have such a moduli space. This leads to the notion of a *stack*, or an *orbifold*.

From the (omitted) definition of a moduli space, statements about a moduli space of your favorite objects translate directly to universal statements about the objects in question. For example, characteristic classes of vector bundles correspond to cohomology classes of classifying spaces.

I am interested in the geometry and topology of moduli spaces, from an algebro-geometric perspective. I will sketch four examples that relate to different sorts of moduli spaces.

In the late nineteenth century, Schubert created a method of solving a wide variety of rather concrete problems, but his methods were not rigorous. He tended to get the right

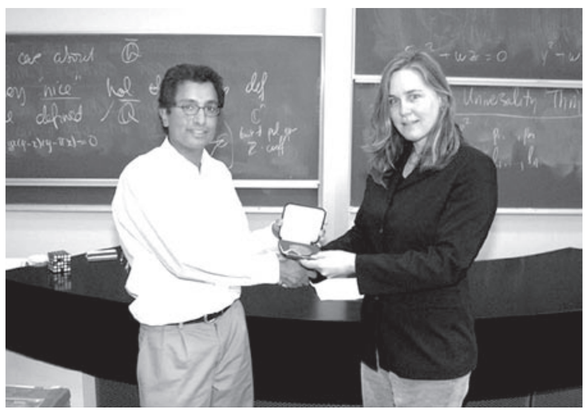
The 2004-2005 André-Aisenstadt Prize

answer, but it wasn't clear why or how; and those following in his footsteps were correct less often. Hilbert's fifteenth problem was in some sense to make precise the intersection-theoretic calculations of Schubert and his school. In a moral sense, this problem was resolved by Fulton and MacPherson, by their construction of *intersection theory* on highly singular (i.e. not smooth) spaces [F]: they gave rigorous and powerful techniques with which we could attempt to tackle these ancient computations. The most difficult calculation was Zeuthen's solution of the characteristic number problems for plane quartic curves; this was a benchmark of our understanding of intersection theory. The hardest of these was the following concrete question: how many degree 4 plane curves are tangent to 14 generally chosen general lines? (There turn out to be a finite number of them — 23,011,191,144 to be precise.) I used a very modern moduli space, the space of *stable maps*, to completely solve the characteristic number problem for quartics. Remarkably, some of Zeuthen's mysterious statements turn out to be interpretable as describing the geometry of this modern space, which came out of string theory. (I was grateful to receive the CMS' G. de B. Robinson award for this article, [VI].) The geometry hidden here turns out to connect a surprising number of other ideas: 12 points on the Riemann sphere, elliptic curves, the E_8 lattice, and more [V2].

I have also studied the geometry and topology of the moduli space of Riemann surfaces (or algebraic curves). Tom Graber and I have used methods from Gromov-Witten theory to show that a large number of conjectures and theorems about this moduli space can be derived from a surprisingly simple and elegant combinatorial statement describing when certain cohomology classes vanish [GrV]. Together with the combinatorialists Ian Goulden and David Jackson, I am using some of these techniques to tackle one of the central conjectures in the field, Faber's intersection number conjecture [GJV].

Another series of papers deals with homogeneous spaces, and in particular the Grassmannian (described above). The topology of the Grassmannian (the "geometry behind linear algebra") has a rich representation-theoretic structure. In order to describe the cohomology ring of the Grassmannian, geometers had to first translate the problem into combinatorics, solve it there, and then translate back to geometry. I gave a naive geometric explanation of the rule in [V4], which led to the solution of a variety of unsolved problems in geometry [V5].

Moduli spaces such as the Grassmannian and the moduli space of Riemann surfaces are very well-behaved; they are smooth, for



Ravi Vakil, 2004-2005 André-Aisenstadt Prize Winner and, Chantal David, CRM Deputy Director

example. A few other moduli spaces that geometers know and use and love are also well-behaved, but many others were not known to be so pleasant, and there has been a great effort trying to bound how bad their behavior can get. Two early examples include the moduli space of algebraic surfaces (e.g. real fourfolds with complex structure), and the parameter space of smooth curves in projective space. In my most recent paper [V6], I show that the situation is much worse than we feared. Many of the moduli spaces we work with on a day-to-day basis are in fact arbitrarily singular: every single singularity appears on them.

These four projects are representative of the type of ideas that have interested me. One of the joys of the field is that I get to work closely with people in other areas, and to learn exciting developments from them. As well as the people mentioned above, I have enjoyed working with Allen Knutson (in representation theory and symplectic geometry), Aleksey Zinger (in differential geometry), Sara Billey (in combinatorics), and Mike Roth (in algebraic geometry), as well as others in topology and mathematical physics.

(*Note: the most recent versions of my articles are available at http://math.stanford.edu/~vakil/preprints.html For a more leisurely introduction to moduli spaces, see the expository article* [V3], *which also announces the main result of* [GrV].)

References

[F] W. Fulton, *Intersection theory*, Springer-Verlag, Berlin, 1998.

[GJV] I.P. Goulden, D.M. Jackson, and R. Vakil, *On Faber's intersection number conjecture*, in preparation.

[GrV] T. Graber and R. Vakil, *Relative virtual localization, and vanishing of tautological classes on moduli spaces of curves*, Duke Math. J., to appear, math.AG/0309227.

[V1] R. Vakil, *Characteristic numbers of quartic plane curves*, Can. J. Math., **51** (1999), no. 5, 1089–1120, math.AG/9812018.

[V2] R. Vakil, *Twelve points on the projective line, branched covers, and rational elliptic fibrations*, Math. Ann., **320** (2001), no. 1, 33–54, math.AG/9910077.

[V3] R. Vakil, *The moduli space of curves and its tautological ring*, Notices of the Amer. Math. Soc. (feature article), **50**, no. 6, June/July 2003, 647–658.

[V4] R. Vakil, *A geometric Littlewood-Richardson rule*, Ann. of Math., to appear, math.AG/0302294.

[V5] R. Vakil, *Schubert induction*, Ann. of Math., to appear, math.AG/0302296.

[V6] R. Vakil, *Murphy's Law in algebraic geometry: Badly-behaved deformation spaces,* submitted for publication, math.AG/0411469.

Analysis in Number Theory by Chantal David (Concordia University) - *continued from page 1*

and informal lectures intended to introduce graduate students and postdoctoral fellows to the topics of the workshop on p-adic representations, organised by Adrian Iovita (Concordia) and Henri Darmon (McGill) from September 12 to September 16, 2005. This workshop explored the p-adic local Langlands correspondence and p-adic families of modular forms, and how those two topics come together. It was the first in a series of workshops dedicated to these themes that will be held around the world in the next years (Palo Alto in February 2006, Luminy in June 2006 and Münster in 2007). The organisers are happy to report that the workshop was quite a success, and that a number of strong and surprising results were announced. Among them we mention a construction of an "inverse correspondence" for the local p-adic Langlands program for $\mathrm{GL}_2(Q_p)$ (P. Colmez), a proof of the Fontaine-Mazur conjecture for p-adic Galois representations which are crystalline at p (M. Kisin) and a proof of Serre's conjecture for odd conductor representations (S. Khare and J.-P. Wintenberger). Furthermore, several collaborations began during the workshop and hopefully will lead to other strong results.

The next workshop, organised by Eyal Goren and Henri Darmon (McGill), explores the relationship between intersection numbers for arithmetic cycles on Shimura varieties, Fourier coefficients of automorphic forms, and special values of L-functions. Spectacular examples include the Gross-Zagier formula for the arithmetic intersection of singular moduli, heights of Heegner points and derivatives of L-functions, the Hirzebruch-Zagier cycles and derivatives of Eisenstein series, elliptic units, as well as recent work in higher dimensional cases. This workshop will be held from December 12 to December 16, 2005. The organisers are also planning a related and informal three-day activity on new directions in Stark's conjectures to be held in December.

During the second term, which will focus on classical analysis and analytic number theory, a workshop on L-functions, organised by Ram Murty (Queens) and Chantal David (Concordia), will be held from February 13 to February 18, 2006. The driving themes for this workshop are: vanishing of L-functions at critical points, zeroes of L-functions, subconvexity estimates for L-functions and applications to equidistribution. The workshop will include three series of lectures by Philippe Michel (Montpellier), Kumar Murty (Toronto) and the André-Aisenstadt chair K. Soundararajan (Michigan), surveying the key ideas and recent progress on L-functions, and presenting the new directions in the field.

The next workshop, organised by Jean-Marie de Koninck (Laval) and Andrew Granville (Montréal), will be held from March 13 to March 17, 2006. In many questions in analytic number theory and its applications, it is necessary to have a good understanding of the internal structure of typical integers, that is of their prime factors, and the interactions between those primes. This has been a central subject for some time, and recent developments are showing that these considerations are also central to the understanding of key questions on character sums and complexity analysis. Several of Paul Erdos' classical questions on those topics have recently been answered by Kevin Ford (Illinois), who will give a series of lectures. One of the goals of the workshop is the emergence of new directions from this work.

The theme year will continue with a concentration period on additive combinatorics. In recent years, results and ideas from additive number theory, harmonic and functional analysis, ergodic theory, combinatorics, probability theory and discrete geometry have been brought together in some spectacular theorems, such as the result of Green and Tao that there are arbitrarily long arithmetic progressions of primes and Bourgain's new bounds for short exponential sums. The concentration period will begin with the CRM-Clay School on Additive Combinatorics, organised by Jozsef Solymosi (UBC), Andrew Granville (Montreal) and David Ellwood (Clay), and held from March 30 to April 5, 2006. The school is intended to give to graduate students, postdoctoral fellows and junior faculty an opportunity to participate in these exciting new developments. A week-long workshop on additive combinatorics will follow immediately. The lecturers of those two events will include Jean Bourgain (IAS, Princeton), Tim Gowers (Cambridge), Ben Green (Bristol), Imre Rusza (Alfred Renyi Institute) and Terry Tao (UCLA). The CRM-Clay School on Additive Combinatorics will be the first school held on this subject in the world.

The last workshop of the theme year is organised jointly with the MSRI, and will be held at the Banff International Research Station (BIRS) on May 13-18, 2006. It is dedicated to the current research highlighting the flow of ideas between analytic number theory and arithmetic geometry. On the analytic side, one has the circle method and its modern adaptations, sieving methods, techniques from spectral theory, ergodic theory and the theory of automorphic forms. As for arithmetic geometry, there is the theory of universal torsors, heights, intersection theory, p-adic integration, theory of moduli spaces, compactifications of algebraic groups and homogeneous spaces. Open problems range from understanding very concrete equations to proving and interpreting asymptotics of rational and integral points on orbits of linear algebraic groups, special points on semi-abelian varieties and Shimura varieties.

There will be a strong emphasis on training during the theme year, whose activities include the SMS-NATO Summer School on Equidistribution in Number Theory in July 2005, and the CRM-Clay School on Additive Combinatorics in April 2006, which are both primarily targeted at graduate students, postdoctoral fellows and junior faculty. Two courses, one per semester, will introduce graduate students to the subjects of the workshops and concentration periods of the theme year. In the Fall, Peter Clark will teach a course on Shimura Varieties, and the course of Andrew Granville in the Winter term will prepare graduate students to the topics of the second term of the theme year. More details about the activities of the 2005-2006 theme year can be found on the web pages of the CRM.

Scientific Reports

Workshop on Algebraic K-theory 2004

October 2-6, 2004

Organizers: Eric Friedlander (Northwestern), Dan Grayson (Illinois, Urbana-Champaign), Rick Jardine (Western Ontario), Manfred Kolster (McMaster)

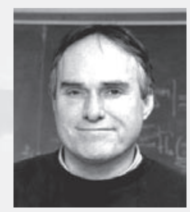
The meeting reflected some of the major developments of the past year in the subject, particularly in motivic homotopy theory. These include Levine's proof of the Voevodsky slice conjecture, Morel's proof of the unstable connectivity theorem for motivic homotopy types (which was described in his talk as a type of Hurewicz theorem), and the identification by Röndigs and Ostvaer of Voevodsky's triangulated category of motives with the stable category of modules over the cycle-theoretic Eilenberg-Mac Lane spectrum. Levine's theorem uses a homotopy theoretic approach to the Chow moving lemma, which was discussed during his talk. The Levine and the Röndigs-Ostvaer results together substantially demystify the relation between the motivic stable category and motivic cohomology, while Morel's work points the way to explicit calculations of motivic homotopy groups. Suslin displayed a spectral sequence for the motivic cohomology of an arbitrary Severi-Brauer variety which is built from a decomposition of the motive associated to its Čech resolution.

It had been expected that the proof of the Bloch-Kato conjecture relating Galois cohomology to the Milnor K-theory of a field would be completely written up by the time of this conference, but this was not to be. Suslin predicts, however, that this proof will be properly written up within a year. The Bloch-Kato conjecture has sensational calculational consequences; these include the Lichtenbaum-Quillen conjecture which says that K-theory can be computed from étale cohomology.

Eric Friedlander

Dan Grayson

Manfred Kolster



Rick Jardine

Report on Stochastic 2004

July 16-26,2004

Organizers: Peter Glynn (Stanford), George Kesidis (The Pennsylvania State University), Donald A. Dawson (McGill), Raj Srinivasan (Saskatchewan)

In July 2004, within the framework of its general program, the CRM hosted a workshop on Stochastic Networks with three different activities. The Madison style Stochastic Network Conference (SNC) included two satellite workshops. The two-day workshop on Economics of Communication Networks preceded the SNC, while the three-day Call Centre Workshop (CCW), partially funded by the Wharton Financial Institutions Center, was at the tail end of the SNC. Here are some comments from each of the three organizers relating to their associated event.

Workshop on Economics of Communication Networks

Org.: George Kesidis

This workshop attracted some of the major figures who are working on increasingly important economic issues related to the theory of communication networking. In particular, two very interesting talks on the role of economic incentives for emerging peer-to-peer networks drew a great deal of discussion during the breaks. Lively discussions also broke out during several talks and participants expressed the value of the clarification of issues that was achieved in this sometimes confusing area where a diversity of issues need to be considered simultaneously. For example,

- the roles and mathematical definitions of « fairness » and how they might be interpreted as regulation in a communication network that is run more according to free market dynamics than in current practice.
- how security, pricing & billing, and quality-of-service may be interrelated.

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SMS-NATO ASI 2004 Session

by Octav Cornea (Université de Montréal)

The 43rd Séminaire de Mathématiques Supérieures (SMS), a NATO Advanced Summer Institute, took place in Montréal from June 21st to July 2nd, 2004. The theme of the meeting was Morse Theoretic Methods in Nonlinear Analysis and in Symplectic Topology. The SMS was organized by Octav Cornea (Montréal) and Paul I. Biran (Tel Aviv). Attended by 65 students as well as by the 12 speakers, it provided an excellent, active audience for the lectures. Octav Cornea comments on its scientific content.

The talks at this meeting centered on Morse theoretical techniques which can be used to solve difficult analytic problems as well as problems in symplectic topology and in robotics.

The key tool in modern symplectic topology is Floer homology techniques, and this has constituted a recurring theme for the lectures given at the SMS.

In the first week Helmut Hofer talked about the foundations of symplectic field theory, one of the major new "machines" in symplectic topology whose development has been pursued by Hofer, Eliashberg and Givental for a number of years now. The origins of symplectic field theory lie in Floer's machinery but it goes much beyond that, and both in applications in complexity. Hofer's lectures were therefore extremely timely. Matthias Schwarz presented his recent proof for a result of Viterbo relating the Floer homology of the cotangent bundle and the string topology of the zero section of this bundle. This notion of string topology has been recently introduced by Chas and Sullivan with a purely topological motivation, and the fact that it fits perfectly with the quantum product in Floer's theory is quite remakable. Michael Farber discussed applications to robotics. Paul Biran described efficient methods to use Floer homology in the monotone case to prove results concerning the topological structure of Lagrangian submanifolds. Leonid Polterovich showed how to relate this symplectic topology to dynamics and geometric group theory methods. Octav Cornea presented higher order Floer type invariants and applications.

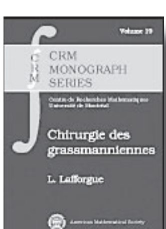
Octav Cornea

This was an intense first week with lectures of the highest order of interest for specialists as well as for beginners in the field. Some of the topics discussed (in particular by Hofer, Schwarz, Cornea and Polterovich) were presented publicly for the first time at the ASI. The second week continued as strongly: Claude Viterbo talked about generating functions techniques, Alberto Abbondandolo discussed Morse theory in Hilbert spaces, Kenji Fukaya talked about a new version of his A^∞ machinery (developed with Oh and Ono) and applications, some of which overlapped with applications obtained by different methods by Cornea jointly with Lalonde and mentioned in the first week. Marek Izydorek lectured on his approach to the infinite dimensional Conley index and Yong-Geun Oh presented his recent spectral invariant techniques and chain level Floer methods. Finally, Ralph Cohen described his topological approach to string topology and its potential implications for symplectic topology.

Obviously, as it follows from the description above, many of the talks of different speakers were strongly interrelated (for example, Hofer's talks and those of Cornea, those of Schwarz with those of Cohen, those of Oh and those of Viterbo). This contributed to the overall quality and strength of the ASI itself.

Les Publications du CRM

Chirurgie des grassmanniennes

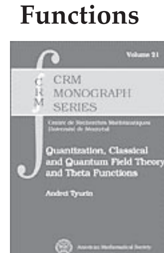


L. Lafforgue, Institut des Hautes Études Scientifiques - AMS, 2003, 170 pp., Hardcover, ISBN 0-8218-3358-8, List: $45, All AMS Members: $36, CRMM/19

Algebraic Structures and Moduli Spaces: CRM Workshop

Jacques Hurtubise, McGill University, and Centre de Recherches Mathématiques, and Eyal Markman, University of Massachusetts, Editors - AMS, 2004, 258 pp., Softcover, ISBN 0-8218-3568-8, List: $79, All AMS Members: $63, CRMP/38

Quantization, Classical and Quantum Field Theory and Theta Functions



Andrei Tyurin, Steklov Institute of Mathematical Sciences - AMS, 2003, 136 pp., Hardcover, ISBN 0-8218-3240-9, List: $43, All AMS Members: $34, CRMM/21

Hecke Algebras with Unequal Parameters

G. Lusztig, Massachusetts Institute of Technology - AMS, 2003, 136 pp., Hardcover, ISBN 0-8218-3356-1, List: $39, All AMS Members: $31, CRMM/18

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The 2004-2005 André-Aisenstadt Chairs

Andrew J. Majda (Courant institute)

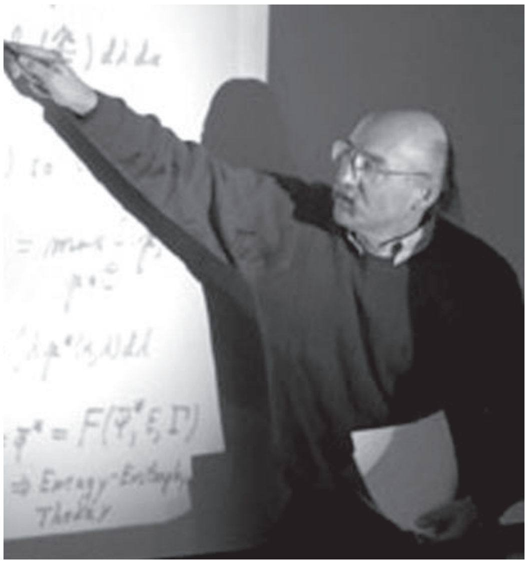
Andrew J. Majda is the Morse Professor of Arts and Sciences at the Courant Institute of New York University. Born in East Chicago in 1949, he received a B.S. degree from Purdue University in 1970 and a Ph.D. from Stanford University in 1973.

Majda's primary research interests are modern applied mathematics in the broadest possible sense, merging asymptotic and numerical methods, physical reasoning and rigorous mathematical analysis. He is well known for both his theoretical contributions to partial differential equations and his applied contributions to diverse areas such as scattering theory, shock waves, combustion, incompressible flow, vortex motion, turbulent diffusion, and atmosphere ocean science.

Majda is a member of the National Academy of Sciences and has received numerous honors and awards including the National Academy of Science Prize in Applied Mathematics, the John von Neumann Prize of the Society of Industrial and Applied Mathematics, and the Gibbs Prize of the American Mathematical Society. He was awarded the Medal of the Collège de France and is a Fellow of the Japan Society for the Promotion of Science. He recently received (2000) an honorary doctorate from his undergraduate alma mater, Purdue University.

He began his scientific career as a Courant Instructor at the Courant Institute from 1973 to 1975. Prior to returning there in 1994, he held professorships at Princeton University (1984-1994), the University of California, Berkeley (1978-1984), and the University of California, Los Angeles (1976-1978). In the past several years at the Courant Institute, Majda has created the Center for Atmosphere Ocean Science with a multi-disciplinary faculty to promote cross-disciplinary research with modern applied mathematics in climate modeling and prediction.

Majda's book with A. Bertozzi, *Vorticity and Incompressible Flow*, was published by Cambridge University Press, and his lecture notes, *Introduction to PDE's and Waves for the Atmosphere and Ocean,* came out recently in the Courant Lecture Notes Series of the AMS. His Aisenstadt lectures at the CRM, written jointly with R.V. Abramov and M.J. Grote, are forthcoming in the CRM Monograph Series of the AMS under the tiltle *Information and Stochastics for Multiscale Nonlinear Systems*.



Andrew J. Majda

Thomas Yizhao Hou (California Institute of Technology)

Thomas Yizhao Hou is the Charles Lee Powell professor of applied and computational mathematics at Caltech, and one of the leading experts in applied and numerical analysis for vortex dynamics and multiscale problems. In his twenty year research career his research interests have been centered around developing analytical tools and effective numerical methods for vortex dynamics, interfacial flows, and multiscale problems.

He was born in Guangzhou, China, and studied at the South China University of Technology before undertaking his Ph.D. at UCLA. Upon obtaining it in 1987, he joined the Courant Institute as a postdoc and then became a faculty member in 1989. He moved to the applied mathematics department at Caltech in 1993, and is currently the executive officer in the department of applied and computational mathematics. He was awarded the Morningside Gold Medal in Applied Mathematics in 2004, the SIAM Wilkinson Prize in Numerical Analysis and Scientific Computing in 2001, the François N. Frenkiel Award from the Division of Fluid Dynamics, American Physical Society in 1998, the Feng Kang Prize in Scientific Computing in 1997, and was a Sloan Foundation Research Fellow from 1990 to 1992. He was also an invited plenary speaker at the International Congress on Industrial and Applied Mathematics in Sydney in 2003, an invited speaker of the International Congress of Mathematicians in Berlin in 1998, and a founding Editor-in-Chief of a SIAM interdisciplinary journal on Multiscale Modeling and Simulation since 2002.

Thomas Yizhao Hou

Rétrospective de l'année thématique 2004-2005

Les mathématiques de la modélisation multiéchelle et stochastique

L'année thématique 2004-2005 sur la modélisation mathématique multiéchelle et stochastique s'achève, et c'est l'heure des bilans et remerciements. Une des missions essentielles du CRM est la formation à la recherche des étudiants et stagiaires postdoctoraux, et l'année qui s'achève a foisonné d'occasions en ce sens, en particulier l'école d'été, pilotée par Eric Vanden-Eijnden, et le cours ISM avancé sous la houlette de Claude Le Bris - deux activités à but pédagogique, certes, mais sans compromis sur la qualité, avec un contenu à un niveau d'excellence qui a réussi à secouer et enthousiasmer les participants, novices et autres.

Anne Bourlioux

Ce bilan est donc une occasion très appropriée de donner la parole à l'un des participants juniors: ci-dessous, le compte-rendu scientifique des activités de l'année du point de vue très personnel de Tony Lelièvre, qui vient de réintégrer son laboratoire au Cermics au terme de son année postdoctorale à Montréal. Une année exceptionnellement productive pour Tony, on peut ajouter: il en a profité, entre les ateliers, école, cours, etc. pour présenter une dizaine de séminaires à Montréal et dans les environs, pour initier des projets de recherche avec divers visiteurs scientifiques, pour finaliser cinq articles, et pour récolter pas moins de trois prix prestigieux pour sa thèse.

Et tout cela, dans une perspective multi-disciplinaire très représentative de la perspective scientifique préconisée tout au long de l'année, puisque ses travaux et présentations ont touché notamment, les simulations de Monte-Carlo quantiques et des questions fines de convergence s'y rapportant, des stratégies de réduction de variance dans les approches stochastiques pour les fluides rhéologiques ainsi que des preuves de convergence du comportement asymptotique en temps de modèles micro-macro pour ces écoulements, le suivi d'interfaces pour la simulation d'instabilités dans les cuves d'électrolyse d'Alcan - tout un programme !

L'heure est aussi aux remerciements: aux organisateurs d'ateliers C. Le Bris, P.Tupper, T.Souganidis, R.Sircar et J.P.Fouque, Weinan E et E. Vanden-Eijnden, en particulier aux deux conférenciers Aisenstadt Andrew Majda et Tom Hou, très généreux dans leurs interactions avec les chercheurs locaux; et finalement, au personnel du CRM pour l'accueil attentif qu'il a réservé aux nombreux participants - les commentaires enthousiastes ont abondé pour souligner à quel point l'organisation locale soignée a favorisé de façon essentielle la qualité des interactions scientifiques.

Anne Bourlioux

Ce petit texte a pour but de donner mes impressions de postdoctorant sur l'année thématique 2004-2005 consacrée aux mathématiques de la modélisation multiéchelle et stochastique. Je me propose tout d'abord de mentionner les divers thèmes abordés dans les conférences de cette année spéciale, puis d'évoquer ensuite plus particulièrement trois ateliers qui m'ont marqué.

Les modèles multiéchelle s'attachent à décrire la réalité, en faisant appel à plusieurs échelles de descriptions: par exemple, dans le domaine de la mécanique des solides, on cherche à comprendre les liens entre les phénomènes à l'échelle de l'atome et les comportements de la matière à l'échelle macroscopique. Ces techniques d'intégration multiéchelle font appel à des technologies scientifiques récentes, dans des domaines très variés.

Les différents cours et ateliers tout au long de l'année ont mis en lumière un domaine où les mathématiciens appliqués jouent un rôle primordial entre les physiciens créateurs de modèle, les mathématiciens motivés par des questions plus théoriques et les numériciens soucieux de construire des algorithmes adaptés au modèle. Une des qualités de ces ateliers était d'ailleurs de réunir des chercheurs de tous ces horizons. Le travail du mathématicien appliqué à l'interface de plusieurs communautés prend toute son importance dans ces nouveaux types de modélisation où des connaissances pluri-disciplinaires sont nécessaires pour progresser.

La variété des techniques mathématiques nécessaires à l'étude de ces modèles (théorie de l'homogénéisation, théorie des larges déviations, théorie ergodique, schémas symplectiques, ...) ainsi que des domaines de la physique concernés (mécanique statistique, modélisation de la turbulence, dynamique moléculaire, climatologie, ...) constituait en soi un attrait majeur de cette année thématique pour un postdoctorant cherchant à ouvrir ses horizons. La conférence organisée par T.Y. Hou donnait un bon exemple de la variété des domaines où la modélisation multiéchelle est utilisée, et de la diversité des techniques mathématiques et numériques nécessaires à l'analyse de ces modèles.

Les modèles aléatoires jouent un rôle particulier dans ces modèles en plein développement, non pas tant par le fait que la meilleure description de la matière à l'échelle la plus microscopique (mécanique quantique) soit intrinsèquement aléatoire, mais plutôt parce que le passage d'une échelle microscopique à une échelle macroscopique consiste à éliminer des degrés de liberté ou à considérer des asymptotiques dans les échelles en temps ou en espace, d'où naît l'aléatoire. L'atelier organisé par A.J. Majda ainsi que ses cours de la Chaire Aisenstadt illustraient particulièrement ces aspects pour des applications à la modélisation climatique, où l'on cherche justement à bâtir, à partir de modèles précis et très complets mais en très grande dimension, des modèles approximés plus simples qui conservent des propriétés statistiques proches du modèle initial, avec pour objectif de comprendre les phénomènes déterminants pour le climat. Les étapes essentielles sont alors la détection des degrés de liberté les plus importants, puis la dérivation d'équations équivalentes sur ces degrés de liberté, en utilisant des relations de fermeture appropriées. Ceci requiert des outils liés à la théorie de l'homogénéisation ou encore à la mécanique statistique. Les modèles aléatoires étaient également au coeur de la conférence consacrée aux mathématiques financières et organisée par R. Sircar et J.-P. Fouque.

L'étude numérique et théorique des couplages de modèles est un autre aspect que ces modélisations multiéchelle ont en commun. En particulier, le couplage de modèles aléatoires et déterministes soulève des questions numériques intéressantes liées aux relations entre les différentes erreurs de discrétisation (en temps, en espace, erreur statistique pour les modèles Monte Carlo). La conférence organisée par A. Bourlioux et P. Souganidis sur les modèles de combustion illustrait particulièrement les questions soulevées par les interactions entre modèles stochastiques (décrivant l'écoulement advectif aléatoire) et modèles déterministes (décrivant la chimie de la combustion). Le couplage de modèles peut aussi se faire à des interfaces, une zone étant décrite plus finement qu'une autre. C'est le problème du passage de l'information aux interfaces qui est alors soulevé. Ce thème était notamment au centre de la conférence organisée par W. E et E. Vanden-Eijnden sur la modélisation multiéchelle dans les solides, où une des questions récurrentes concernait le choix des conditions aux limites entre les modèles de dynamique moléculaire essentiellement discrets, et les modèles de mécanique des milieux continus. Ce domaine est un exemple où il reste encore beaucoup de questions théoriques pertinentes, sur des modèles simplifiés (comportement d'une chaîne de ressort uni-dimensionnel par exemple): Quelle notion de minimum (locale ou globale) de l'énergie mécanique faut-il considérer ? Quelle est la bonne dynamique ?

Certaines conférences ont particulièrement retenu mon attention. Le cours d'introduction au calcul stochastique d'E. Vanden-Eijnden permettait d'introduire des notions récurrentes dans cette année thématique, intervenant par exemple en dynamique moléculaire. Les simulations en dynamique moléculaire, à une échelle intermédiaire entre la mécanique quantique et la mécanique des milieux continus, occupent en effet une place prépondérante dans les modèles multiéchelle, et l'analyse mathématique de ces simulations nécessite des outils probabilistes avancés (théorie des larges déviations, théorie ergodique, étude des processus stochastiques sur une variété). L'atelier sur la dynamique moléculaire de P.F. Tupper et A.M. Stuart a constitué pour moi une superbe introduction à ce domaine dans lequel beaucoup de questions restent posées au mathématicien.

Les problématiques de la dynamique moléculaire (dynamiques contraintes, ergodicité, calcul d'énergies libres, ...) vont certainement influencer mon activité de recherche future. En collaboration avec E. Vanden-Eijnden et C. Le Bris, nous avons d'ores et déjà obtenu des résultats concernant l'analyse numérique de schémas de discrétisation des équations différentielles stochastiques avec contrainte. Ce type de question apparaît notamment en dynamique moléculaire dans le calcul d'énergie libre en fonction des coordonnées de réaction, ou encore pour la simulation des polymères rigides en solution (cristaux liquides).

Il me reste à évoquer la conférence organisée par C. Le Bris sur les modèles rhéologiques multiéchelle, qui a constitué une première pour moi: c'est en effet la première conférence au cours de laquelle j'ai pu suivre et comprendre l'ensemble des exposés, étant donnée qu'elle portait sur le sujet même de ma thèse ! Cette conférence a d'ailleurs été l'occasion de conclure un travail amorcé durant ma thèse après une discussion décisive avec F. Otto, sur l'étude du comportement en temps long d'un système micro-macro pour les fluides polymériques par des techniques d'entropie (cf. preprint CRM-3188).

Cette année au Centre de Recherches Mathématiques m'a donc été très profitable. Je voudrais terminer ce compte-rendu très informel et personnel en remerciant Anne Bourlioux pour l'organisation de cette année thématique et nos nombreuses randonnées, les membres de l'administration du CRM pour leur accueil, Michel Delfour pour son support financier, Stéphane Dellacherie pour nos longues discussions scientifiques ou autres, et Eric Vanden-Eijnden avec qui j'ai commencé une collaboration fructueuse au cours de cette année.

Tony Lelièvre

Tony Lelièvre

À propos des boursiers postdoctoraux

Alain Bourget, stagiaire postdoctoral du CRM à McGill en 2004, enseigne présentement à la University of Massachusetts à Amherst.

Guillaume Bourque, boursier postdoctoral du CRSNG au CRM de 2002 à 2004, a obtenu un poste de chercheur à Singapoure. Pendant son stage au CRM, M. Bourque a conçu un algorithme de séquençage de génome.

Pietro-Luciano Buono, boursier postdoctoral au CRM de 2001 à 2004 (dont deux ans comme boursier CRSNG), est professeur adjoint en mathématiques à la University of Ontario Institute of Technology, à Oshawa. M. Buono se spécialise en théorie des bifurcations.

Marcelo Lanzilotta Mernies, boursier postdoctoral CRM-ISM en 2002-2004 à l'Université de Sherbrooke, est maintentnat professeur adjoint à la Universidad de la Republica de l'Uruguay et chercheur au Centre de mathématique de la même institution. M. Lanzilotta a travaillé sur les méthodes homologiques en théorie des représentations des algèbres pendant son stage.

Trois prix sont venus couronner la thèse de **Tony Lelièvre**, qui a passé l'année 2004-2005 comme stagiaire postdoctoral au CRM dans le cadre de l'année thématique sur les mathématiques de la modélisation multiéchelle et stochastique. Effectuée au CERMICS de l'Ecole Nationale des Ponts et Chaussées (ENPC) sous la direction de Benjamin Jourdain et Claude Le Bris, cette thèse portait sur les *Modèles multi-échelles pour les fluides viscoélastiques*. Elle a valu à son auteur le prix de thèse 2004 de la Fondation de l'ENPC, le prix de thèse Paris Tech placé sous le haut patronage de l'Académie des Technologies et finalement le premier prix de thèse GAMNI (Groupe pour l'Avancement des Méthodes Numériques de l'Ingénieur).

Ambrus Pal, boursier postdoctoral CRM-ISM de 2001 à 2003 et stagiaire postdoctoral du CICMA en 2003-2004, poursuit ses recherches postdoctorales à l'IHES en France.

Yan Pautrat, stagiaire postdoctoral du CRM à McGill en 2003-2004, est maître de conférence à l'Université Paris-Sud, Orsay. Ses travaux portent sur le calcul stochastique quantique et ses applications.

Alexei Penskoi, stagiaire postdoctoral du groupe de physique mathématique du CRM de 2001 à 2004, est chargé de cours (Lecturer) à l'Université indépendante de Moscou et professeur adjoint à l'Université d'État Technique Bauman de Moscou.

Anupam Saikia, boursier postdoctoral CRM-ISM de 2002 à 2004, est professeur adjoint au Département de mathématiques du Indian Institute of Technology de Bombay. Les travaux de recherche de M. Saikia sont en théorie des nombres.

Dr. Libor Snobl arrived in Montreal in February 2004 as a postdoctoral fellow supported by the Mathematical Physics Laboratory. He has been working very productively with two members of that Laboratory. His collaboration with A.M. Grundland concerns the embedding of surfaces associated with certain types of sigma models on Minkowski space into *su(n)* Lie algebras. The surfaces have interesting geometrical properties that should lead to physical applications. Libor's work with P. Winternitz concerns an unsolved problem in Lie algebra theory, namely the classification of solvable Lie algebras and more generally, the basis free identification of arbitrary finite dimensional Lie algebras. The productivity of his first year in Montreal is indicated by his high publication rate: 5 articles published or accepted, one more submitted.

Shannon Starr, boursier postdoctoral CRM-ISM en 2004, est professeur auxiliaire à UCLA depuis janvier 2005. Il a également complété un stage postdoctoral d'un an avec le groupe de physique mathématique du CRM en 2003-2004.

Piergiulio Tempesta, boursier CRM-ISM de 2001 à 2003 et stagiaire du groupe de physique mathématique en 2003-2004, est chercheur au SISSA de Trieste.

Dimiter Vassilev, boursier CRM-ISM de 2003 à 2004 au CIRGET à l'UQAM, est présentement professeur invité à la University of Arkansas.

Dr. Ismet Yurdusen arrived in Montreal in February 2005, shortly after receiving his PhD degree from the Middle East Technical University in Ankara, Turkey. His visit was made possible by a 6 month fellowship that was awarded by the Scientific and Technical Research Council of Turkey. Immediately after his arrival he began a collaboration with P. Winternitz on integrable and superintegrable quantum mechanical systems involving particles with spin. So far they have shown that such systems exist and are physically interesting. The classification of integrable and superintegrable systems with spin is in progress. Previous studies of integrability in classical and quantum mechanics were restricted to scalar particles.

Alexander Zhalij, boursier CRSNG-OTAN au CRM de 2002 à 2004, est chercheur adjoint de l'Institut de mathématiques de l'Académie nationale des sciences d'Ukraine à Kiev.

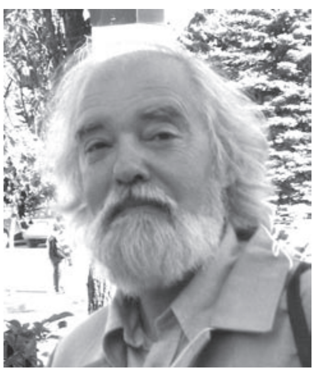
Nouvelles Nouvelles Nouvelles

Le prix de recherche 2005 de la SCMAI, le Grand Prix, et les mathématiques du pneu

La Société canadienne de mathématiques appliquées et industrielles (SCMAI) a décerné son prix de la recherche[1] 2005 à Michel Fortin de l'Université Laval pour ses contributions aux méthodes d'éléments finis mixtes et à la mécanique des fluides, et leurs applications à la modélisation numérique des grands problèmes industriels.

Son exposé à Winnipeg a porté sur les mathématiques et le numérique du pneu auxquels il travaille pour Michelin. Son exposé a eu lieu le jeudi 16 juin, le prix lui a été décerné le vendredi 17 juin au banquet de la SCMAI au Fort Gibraltar, et le samedi 18 juin le pneu de Michelin explosait à Indianapolis confortant du même coup les défis évoqués par Fortin dans les mathématiques du pneu.

Michel Fortin est originaire de Québec. Il obtient son doctorat d'état français sous la direction de Roger Temam et du regreté Jacques-Louis Lions ce qui en fait probablement le seul "lionceau" canadien. Sa thèse portait sur la simulation numérique en mécanique des fluides par la méthode des éléments finis avec application aux fluides de Bingham. Pour attaquer ces problèmes, il eut à employer une panoplie de techniques d'origines diverses (éléments finis, programmation mathématique, analyse convexe) à une époque où les méthodes d'éléments finis pour les écoulements incompressibles étaient un domaine encore vierge. Fortin développa les premiers éléments discontinus en pression. Le lien avec les méthodes d'éléments finis mixtes devint plus précis dans les années qui suivirent et menèrent à ce qui est largement connu comme le critère de Fortin qui, dans plusieurs cas simples, coïncide avec la condition inf-sup de Babuska-Brezzi. Ce travail mena à une collaboration durable avec Franco Brezzi, d'où émergea un livre, *Mixed and hybrid finite element methods*, ainsi qu'à plusieurs articles de recherche et à son implication dans une foule de projets comme la navette européenne Hermes, ou encore sa collaboration avec Pratt et Whitney et, tout récemment, Michelin.



Michel Fortin

Sa capacité de faire des contributions de tout premier plan à la recherche fondamentale en mécanique des fluides et aux mathématiques computationnelles et de les appliquer avec succès aux problèmes urgents d'un intérêt névralgique pour l'industrie, sa fondation du GIREF à Laval, son étonnante diversité intellectuelle, et son approche prudente, disciplinée et cependant visionaire au développement des connaissances ont fait de lui un leader exceptionnel et recherché sur la scène canadienne et internationale.

[1]Ce prix a été établi en 2003 par la SCMAI pour reconnaître des contributions innovatrices et exceptionnelles à la recherche dans un domaine en émergence des mathématiques appliquées ou industrielles. Il consiste en un chèque de 1,000$, une allocation de voyage de 500$, et une plaque commémorative qui sont présentés à la réunion annuelle de la SCMAI où le récipiendaire est invité à prononcer une conférence plénière. Les lauréats précédents du prix ont été Jianhong Wu (York University) en 2003 et Robert Russell (Simon Fraser University) en 2004.

Le 1er septembre 2005, le professeur **Jiri Patera** recevait un doctorat honoris causa de l'Université technique tchèque à Prague. Membre du CRM depuis 1969, il est professeur au Département de mathématiques et de statistique de l'Université de Montréal depuis 1983. Ses contributions exceptionnelles portent sur la théorie des groupes et des algèbres de Lie et leurs applications à la physique des particules et à la chimie quantique. Plus récemment, il s'est penché sur la théorie mathématique des quasicristaux et ses applications à la physique de l'état solide et à la cryptographie. En 2004, Jiri Patera recevait le Prix ACP-CRM de physique théorique et mathématique.

Jiri Patera

François Lalonde, directeur du Centre de recherches mathématiques, prononcera une conférence invitée au Congrès international des mathématiciens (ICM) de 2006 qui se tiendra à Madrid. Des travaux plus anciens avaient été présentés par sa collaboratrice Dusa McDuff en plénière du ICM 1998.

Par ailleurs, un atelier organisé à la mi-octobre 2005 par l'Institute for Advanced Study (Princeton) abordera la nouvelle théorie de Cornea-Lalonde sur le "Cluster complex" (théorie universelle de Floer) et ses relations avec la théorie des "polyfolds" de Hofer pour les espaces de modules généraux.

Le 1er juin dernier, le nouveau recteur de l'Université de Montréal, Luc Vinet, entrait en fonction pour un mandat de 5 ans.

Monsieur Vinet était auparavant vice-principal exécutif de l'Université McGill. Détenteur d'un Ph.D. en physique théorique de l'Université de Montréal et professeur au Département de physique de l'Université de Montréal, il a été directeur du Centre de recherches mathématiques de 1994 à 1999, et président du Réseau de calcul et modélisation mathématique (rcm_2). Depuis 1999, il était professeur titulaire au Département de mathématiques et statistique et au Département de physique de l'Université McGill.

Luc Vinet

Research Contributions of the 2005 CRM-SSC Award Winner

by Jiahua Chen (University of Waterloo)

The CRM-SSC Prize in Statistics was awarded this year to Jiahua Chen (Department of Statistics and Actuarial Science, University of Waterloo) for his outstanding contributions to the statistical sciences. Professor Chen was born in 1960 in the People's Republic of China. In spite of a difficult youth, he majored in mathematics in 1982 at the University of Science and Technology of China. After doing his M. Sc. at the Institute of Systems Science of the Academia Sinica (1982), he obtained a Ph. D. from the University of Wisconsin-Madison in 1990. He discusses here his main research interests.

To many people's dismay, in spite of rapid advances in information technology, it is still not always possible to make *precise* predictions when dealing, for instance, with the evolution of the stock market or the outcome of a medical treatment. Under such conditions, instead of demanding zero-error predictions, we accept random errors as part of life and, consequently, adopt a two-step strategy with regard to them. At first we eliminate part (if not all) of them, and subsequently shed light on the remaining ones. We thus design our experiments so as to limit the influence of random errors, and then fit a statistical model to the experimental data along with an assessment of uncertainty. This is primarily what statisticians do and advise people to do.

In statistical inference, it is often assumed that the data collected can be properly fitted by a distribution in a family of probability distributions. The objective is to find which one amongst the distributions in the chosen family fits the data best. Often we may first have to address the question of model selection: does the family of distributions give us enough choices for fitting, or too many choices, or too complicated ones, so that a solution is hard to pin down? In practice, we usually prefer a parametric family in which the distributions can be indexed smoothly by some real parameters. There are two basic considerations in choosing a model: it should (1) possess simple mathematical properties and (2) describe accurately the problem under study. The first requirement is a technical one, subject to the second. For example, the Binomial model with parameter p (a real number between 0 and 1) is a good model to fit the number of successes observed from a fixed number of independent Bernoulli trials. If 30 successes in 100 trials are observed, the parameter p is often estimated as 30/100. One may further wonder whether the "true" value of the parameter p is actually as large as $p = 0.5$, the observed 30/100 being only an unlucky event. A simple calculation indicates that if indeed $p = 0.5$, we would have been extremely unlucky. The chance of having observed 30 or fewer successes when $p = 0.5$ is about 0.00004, so the reality of 30 successes would be unlikely and wondering whether $p = 0.5$ is unnecessary.

Inference methods for classical regular statistical models such as the binomial model are well developed. Part of my research in recent years is about a class of non-regular, finite mixture models. Let $f(x;\theta)$ be a probability density function where θ is a vector of real valued parameters. A finite mixture distribution with K components and the component probability density function $f(x;\theta)$ has a probability density function of the form

$$f(x;G) = \pi_1 f(x;\theta_1) + \pi_2 f(x;\theta_2) + \cdots + \pi_K f(x;\theta_K) = \int f(x;\theta)dG(\theta),$$

where $\pi_j > 0$, $\sum_{j=1}^K \pi_j = 1$, and G is the mixing distribution defined as

$$G(\theta) = \sum_{j=1}^K \pi_j I(\theta_j \le \theta),$$

with $I(\theta_j \le \theta)$ being an indicator function. Consider the example when $f(x;\theta)$ is a binomial distribution with success probability θ. A binomial mixture model with $K = 2$ indicates that the observed population consists of two different, but unidentified binomial subpopulations: each independent observation has probability π_1 of being from one binomial subpopulation, with probability π_2 of being from the other. For example, one binomial subpopulation may consist of families possessing a disease gene linked to the marker under investigation, while the other consists of families possessing an unlinked disease gene [1]. Genetic researchers wish to know whether the observed population indeed contains families with a linked disease gene or not. Unlike the ordinary (non-mixture) binomial distribution, the binomial mixture model is non-regular in the sense that it does not satisfy the regularity conditions for standard statistical procedures.

In general, one assumes that one has n observations from a mixture model $f(x; G)$ with $K = 2$. A statistical question which arises is whether there is enough evidence to reject the hypothesis that $\theta_1 = \theta_2$, i.e., that K can be reduced from 2 to 1 in the model expression, in favour of $\theta_1 \neq \theta_2$, i.e., K can't be reduced from 2 to 1. A classical approach is as follows. Define the log-likelihood function

$$l_n(G) = \sum_{i=1}^n \log f(x_i; G),$$

where x_i, $i = 1, \ldots, n$ is a random sample. Let $\hat{G}_0$ and $\hat{G}_1$ be the maximum points of $l_n(G)$ under the assumptions of $K = 1$ and $K = 2$ respectively. The likelihood ratio test (LRT) statistic is defined as

$$R_n = 2\{l_n(\hat{G}_1) - l_n(\hat{G}_0)\}.$$

We would reject the model with $K = 1$ in favour of $K = 2$ when R_n exceeds a pre-set threshold. The difficult problem arising now is that of determining a threshold with a given significance level.

For regular parametric models, the null distribution of R_n is well approximated by a chi-squared distribution when n is reasonably large, so that an asymptotic threshold can be easily read from a chi-squared distribution table. For finite mixture models, the use of a chi-squared threshold will result in a substantially excessive number of false rejections. There may be two approaches to correct this. One approach is to find the correct threshold. Unfortunately this is not an easy task. It usually entails calculating a quantile of the supremum of a chi-squared process [2, 3, 4, 5, 6] which differs from one mixture model to another. My research has led to another approach: modify the likelihood function so that the modified LRT statistic has a limiting distribution with which one can easily calculate the quantiles, with a negligible compromise in the testing power compared to the ordinary LRT. In my paper [7], the following modified likelihood function is proposed:

$$\tilde{l}_n(G) = l_n(G) + C\sum_{j=1}^{K} \log \pi_j,$$

where $C > 0$ is a constant controlling the size of the penalty. Let $\tilde{G}_0$ and $\tilde{G}_1$ be the maximum points of $\tilde{l}_n(G)$ under the assumptions $K = 1$ and $K = 2$ respectively. The modified likelihood ratio test statistic is defined as

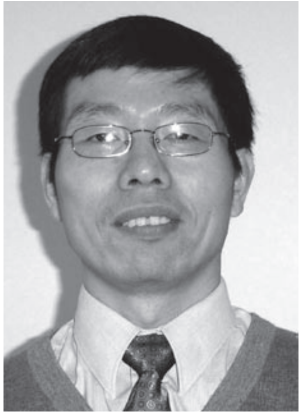
$$\tilde{R}_n = 2\{\tilde{l}_n(\tilde{G}_1) - \tilde{l}_n(\tilde{G}_0)\}.$$

It is shown that $\tilde{R}_n$ is well approximated by a mixture of chi-squared distributions and that its testing power is comparable with the ordinary LRT. In the series of papers with my collaborators [8, 9, 10], this idea is further explored and found to be widely applicable, and it is shown that the modified LRT provides simple, but efficient solutions to many problems related to finite mixture models.

My other research interests include statistical genetics, empirical likelihood, sampling survey, and so on. I would like to say a few words on my contributions to sampling survey. In sampling survey, nonresponse cases (missing data) happen almost all the time. Often the missing data are replaced with an estimate via an imputing method, so that a standard statistical procedure can be used to analyze the survey data. A very appealing imputing scheme is to replace a missing response with the response of a most similar individual. The problem is how to incorporate the additional uncertainty. Due to the non-smooth nature of the imputation procedure, a reasonable account of assessments of the procedure had been lacking for a long time. In joint papers with J. Shao [11, 12], we provided rigorous mathematical assessments of the bias and standard deviations of a class of commonly-used estimators and designed some data-based resampling algorithms for computing the bias and standard deviations. The results provide very useful tools for practitioners of sampling survey.

References

1. Ott, J. (1999). *Analysis of Human Genetic Linkage,* Third Edition. Baltimore: The Johns Hopkins University Press.
2. Chen, H. and Chen, J. (2001). *The likelihood ratio test for homogeneity in finite mixture models.* Canadian Journal of Statistics, **29** 201-215.
3. Chen, H. and Chen, J. (2003). *Tests for homogeneity in normal mixtures in the presence of a structural parameter.* Statistica Sinica, **13** 351-365.
4. Chernoff, H. and Lander, E. (1995). *Asymptotic distribution of the likelihood ratio test that a mixture of two binomials is a single binomial.* Journal of Statistical Planning and Inference, **43** 19-40.
5. Davies, R. B. (1987). *Hypothesis testing when a nuisance parameter is present only under the alternative.* Biometrika, **74** 33-43.
6. Liu, X. and Shao, Y. (2003). *Asymptotics for likelihood ratio tests under loss of identifiability.* Annals of Statististics, **31** 807-832.
7. Chen, J. (1998). *Penalized likelihood-ratio test for finite mixture models with multinomial observations.* Canadian Journal of Statistics, **26** 583-599.
8. Chen, H., Chen, J. and Kalbfleisch, J. D. (2001). *A modified likelihood ratio test for homogeneity in finite mixture models.* Journal of the Royal Statistical Society B, **63** 19-29.
9. Chen, H., Chen, J. and Kalbfleisch, J. D. (2004). *Testing for a finite mixture model with two components.* Journal of the Royal Statistical Society B, **66** 95-115.
10. Chen, J. and Kalbfleisch, J. D. (2004). *Modified likelihood ratio test in finite mixture models with a structural parameter.* Journal of Statistical Planning and Inference, **129** 93-107.
11. Chen, J. and Shao, J. (2000). *Nearest neighbor imputation for survey data.* Journal of Official Statistics, **16** 113-131.
12. Chen, J. and Shao, J. (2001). *Jackknife variance estimation for nearest neighbor imputation.* Journal of American Statistical Association, **96** 260-269.



Jiahua Chen

CRM-Fields Prize 2005 Recipient

Explicit formulas for Mahler measure

by David W. Boyd

The CRM-Fields Prize for 2005 was awarded to Professor David Boyd in recognition of his exceptional work in analytic number theory. After receiving his B.Sc. from Carleton University in 1963 and his Ph.D. from the University of Toronto in 1966, Professor Boyd taught at the University of Alberta and at Caltech. In 1971 he joined the staff at the University of British Columbia, where he is currently Full Professor. He is a winner of the E.W.R. Steacie Prize, a Fellow of the Royal Society of Canada, and has won both the Canadian Mathematical Society's Coxeter-James and Jeffery-Williams Prize Lectures. His service to the Canadian mathematical community includes terms as vice-president of the CMS, chair of the NSERC Mathematics grant selection committee, and Acting Director of PIMS. Here is how he outlines the main themes of his research.

Mahler introduced his eponymous measure as a tool in transcendence theory. Given a polynomial $P(x_1, \ldots, x_n)$, one defines its logarithmic Mahler measure by

$$m(P) = \int_0^1 \cdots \int_0^1 \log |P(e(t_1), \ldots, e(t_n))| dt_1 \ldots dt_n, \tag{1}$$

where $e(t) := \exp(2\pi i t)$, so that $m(P)$ is the average of $|P|$ over the real n-torus. In fact Mahler defined $M(P) = \exp(m(P))$ which is the geometric mean of $|P|$ over the torus but, for our purposes, $m(P)$ is the more natural object to study. Quantities of the form $m(P)$ often appear as rates of growth of discrete dynamical systems. In fact, a result of Lind, Schmidt and Ward [11] shows that for every $P(x_1,\ldots,x_n)$ with integer coefficients, $m(P)$ is the entropy of a dynamical system called a $\mathbb{Z}^d$-action.

For one-variable polynomials, Jensen's formula gives

$$m(P) = |a| \sum_\alpha \log \max(|\alpha|, 1), \tag{2}$$

where a is the leading coefficient and x a typical root of P. It is natural to ask for similar formulas for the measure of polynomials in many variables. The first such formula was due to Smyth [2, 15] who showed that

$$m(1 + x + y) = \frac{3\sqrt{3}}{4\pi} L(\chi_{-3}, 2), \tag{3}$$

where $L(\chi, s) = \sum_{n=1}^\infty \chi(n) n^{-s}$ is a Dirichlet L-series for the character $\chi_{-3}(n) = \left(\frac{-3}{n}\right)$.

Although a few other formulas like (3) involving Dirichlet L- series were discovered in the intervening years, there was not much progress until Deninger's analysis of the question in terms of motivic cohomology and his evaluation of $m(x + y + 1 + 1/x + 1/y)$ as a Kronecker-Eisenstein series for an elliptic curve E of conductor 15 [8]. This formula led him to conjecture that

$$m(1 + x + y + 1/x + 1/y) \stackrel{?}{=} \frac{15 L(E, 2)}{4\pi^2} = L'(E, 0), \tag{4}$$

where $L(E,s)$ is the L-series of an elliptic curve of conductor 15, a result which has been verified numerically to 100 decimal places.

Deninger's result led me to experimentally study families of polynomials defining elliptic curves, e.g. $P_k(x, y) = k + x + y + 1/x + 1/y$ and to see if similar formulas would hold for these. The experiments described in [3] led to many such examples and to conjectural conditions on the polynomials $P(x, y)$ in order that a formula of the type $m(P) = rL'(E, 0)$ would hold for the L-function of an elliptic curve and a rational r. Rodriguez Villegas [13] developed a new approach to derive Kronecker-Eisenstein formulas for the families considered in [3] and showed that the conjectures given in [3] were consequences of the Bloch-Beilinson conjectures. His paper makes explicit the connection with K-theory. The K-group that matters here is $K_2(E)$.

In another direction, there is a class of 2-variable polynomials for which one can compute $\pi m(P(x, y))$ as a sum of values of the Bloch-Wigner dilogarithm evaluated at algebraic numbers. This is an exact analogue of the basic formula (1). These polynomials are the so-called A-polynomials (not Alexander polynomials) of hyperbolic manifolds [4]. The terms in the sum have a geometric meaning. They can be grouped into terms which correspond to various *pseudo-triangulations* of the manifold into unions of ideal tetrahedra. One of these sums gives the volume of the manifold. In a few cases, this is the only term present and $\pi m(P(x, y))$ is exactly the volume of the manifold, but this is quite unusual. If this happens, we obtain formulas of the form

$$\pi m(P(x, y)) = r|D|^{3/2} \zeta_F(2)/(2\pi)^{2d-2}, \tag{5}$$

where F is an algebraic number field of degree d with exactly one complex embedding and with discriminant D, and where r is a rational number, analogous to Smyth's formula (3).

An interesting feature of formulas like (5) is that, although r is known to be rational and can be computed to arbitrary accuracy, we are rarely able to prove that r is the rational it seems to be. But it should be emphasized that the equality between $\pi m(P(x, y))$ and the volume in these cases does not have this defect. It is only in relating the volume to the zeta function that this problem arises. In [5] we produce a large number of identies like (5) for a number of quadratic and quartic fields without using hyperbolic geometry. In [6] we explore situations in which one can use the connection with hyperbolic geometry to completely prove some formulas of this type.

For polynomials in more than 2 variables, up to very recently, the only example one had of formula like (3) was again due to Smyth [15], namely

$$\pi^2 m(1 + x + y + z) = \frac{7}{2} \zeta(3). \tag{6}$$

Recently Lalín [9], [10] has proved many formulas of a similar sort for polynomials of higher degrees, one of the most striking being the following 5-variable example

$$\pi^4 m((1+u)(1+v)(1+x)+(1-u)(1-v)(y+z)) = 93\zeta(5). \quad (7)$$

One would also expect formulas analogous to (4) for higher dimensional varieties but there are few of these yet known. Recently, Bertin [1] has proved some 3-dimensional generalizations of the formulas of Rodriguez-Villegas for some families of polynomials defining Calabi-Yau three folds. Among the many results in her paper is the following

$$\pi^3 m(6+x+y+z+1/x+1/y+1/z) = 24\sqrt{6}L_F(\phi,3), \quad (8)$$

where $F = \mathbb{Q}(\sqrt{-6})$, and $L_F(\phi,s)$ is a Hecke L-series.

At the BIRS workshop on Mahler measure in 2003, Vincent Maillot [12] gave an intriguing analysis of the Mahler measure of non-reciprocal polynomials, i.e., polynomials for which $P(1/x_1,\ldots,1/x_n)/P(x_1,\ldots,x_n)$ is not a monomial. In this case the intersection of $P(x_1,\ldots,x_n)=0$ with the torus $|x_1| = \cdots = |x_n| = 1$ is an algebraic variety of dimension less than n and the value of the Mahler measure should be determined by the cohomology of this variety.

Rodriguez Villegas followed this suggestion up for some specific examples and was led to conjecture explicit values for the next two terms in the sequence starting (3), (6), namely

$$m(1+x_1+x_2+x_3+x_4) \stackrel{?}{=} L'(f,-1), \quad (9)$$

where f is a cusp form of weight 3 and conductor 15, and

$$m(1+x_1+x_2+x_3+x_4+x_5) \stackrel{?}{=} 4L'(g,-1), \quad (10)$$

where g is a cusp form of weight 4 and conductor 6. (Note that, by the functional equations for these forms, the quantities in (9) and (10) are multiples of $L(f,4)$ and $L(g,5)$). He verified these results by computation to a convincing number of decimal places. Sadly, the conjectures do not extend to more variables since the appropriate space of cusp forms has dimension greater than 1 once the number of variables is more than 5.

Rodriguez Villegas and I have also explored situations in which Maillot's analysis leads us to expect that the Mahler measure of certain 3-variable polynomials should be related to the values $L(E,3)$ for certain elliptic curves. We obtained five specific examples, variants on the following

$$m((1+x)(1+y)+z) \stackrel{?}{=} 4\Big(\frac{\sqrt{15}}{2\pi}\Big)^4 L(E,3) = 2L'(E,-1), \quad (11)$$

where E is an elliptic curve of conductor 15.

In one of the examples we considered, the variety in question degenerated from an elliptic curve to a curve of genus 0 leading to the conjecture that

$$\pi^2 m(1+x+(1-x)(y+z)) = \frac{28}{5}\zeta(3). \quad (12)$$

In (12) $\stackrel{?}{=}$ has been written as = since this is now a theorem due to Condon [7]. (The proof is not easy!)

The recent rapid progress that has been made is due to a combination of theoretical insight and numerical experimentation. Neither by itself would be likely to produce the sort of results we have described. For example, in the recent conjectures (9), (10), it is not easy to produce highly accurate numerical values for the measures that appear. After all, these are 4 and 5 dimensional integrals. But fortunately, Rodriguez Villegas, Toledano and Vaaler [14] had worked out special methods for linear polynomials which enabled them to obtain about 15 decimal place accuracy for the numbers appearing in the left member of (9) and (10). This is not nearly enough accuracy to employ the usual paradigm of guessing some of the ingredients that might appear in an explicit formula and then employing the LLL or PSLQ algorithms. However, in this case the insight provided by Maillot's analysis enabled Rodriguez Villegas to guess the exact L-function values that would appear and a simple division convincingly identified the constants as being 1 and 4 respectively.

It is a major challenge to completely understand and give complete proofs of the formulas that have recently been discovered. And no doubt there are many more fascinating formulas left to discover.

References

[1] M.J. Bertin, *Mesure de Mahler d'hypersurfaces K3*, ArXiv math NT/0501153, January 11, 2005.

[2] D.W. Boyd, *Speculations concerning the range of Mahler's measure*, Canad. Math. Bull., **24** (1981), 453–469.

[3] D.W. Boyd, *Mahler's measure and special values of L-functions*, Experiment. Math. **37** (1998), 37–82.

[4] D.W. Boyd, *Mahler's measure and invariants of hyperbolic manifolds*, in Number Theory for the Millenium, ed. M.A. Bennett et al, A.K. Peters, 2001, 127–143.

[5] D.W. Boyd and F. Rodriguez-Villegas, *Mahler's measure and the dilogarithm (I)*, Canad. J. Math., **54** (2002), 468–492.

[6] D.W. Boyd, F. Rodriguez-Villegas, and N. Dunfield, *Mahler's measure and the dilogarithm (II)*, ArXiv math NT/0308041, August 5, 2003.

[7] J. D. Condon, *Mahler measure evaluations in terms of polylogarithms*, Ph.D. thesis, U. Texas at Austin, 2004.

[8] C. Deninger, *Deligne periods of mixed motives, K-theory and the entropy of certain $\mathbb{Z}^n$-actions*, J. Amer. Math. Soc., **10** (1997), 259–281.

[9] M.N. Lalín, *Some examples of Mahler measures as multiple polylogarithms*, J. Number Theory, **103** (2003), 85–108.

[10] M.N. Lalín, *Mahler measure of some n-variable polynomial families*, J. Number Theory, to appear (available online April 30, 2005).

[11] D. Lind, K. Schmidt and T. Ward, *Mahler measure and entropy for commuting automorphisms of compact groups*, Invent. Math., **101** (1990), 593–629.

[12] V. Maillot, Unpublished lecture at the BIRS workshop, *The Many Aspects of Mahler's Measure*, April 26 –May 1, 2003.

[13] F. Rodriguez Villegas, *Modular Mahler measures I, in Topics in Number Theory*, ed. S.D. Ahlgren et al, Kluwer, 1999, 17–48.

[14] F. Rodrigez-Villegas, R. Toledano and J. Vaaler, *Estimates for Mahler's measure of a linear form*, Proc. Edinb. Math. Soc. (2) **47** (2004), 473–494.

[15] C.J. Smyth, *On measures of polynomials in several variables*, Bull. Austral. Math. Soc., **23** (1981), 49–63.

Short Program on Riemannian Geometry

(CRM, June 28th – July 16th 2004)
by Vestislav Apostolov (Université du Québec à Montréal)

This three-week program, held during the summer of 2004 at the Centre de recherches mathématiques, focused on the study of Riemannian metrics whose curvature satisfies constraints (the so-called special geometries). The relation between curvature and topology has been of interest since the beginning of differential geometry and, more recently, metrics with special curvature properties have come to the fore in physical problems related to string theory. These subjects were the leitmotivs of the program that was attended by over 90 participants.

The program opened with a week of introductory short courses designed for graduate students and postdoctoral fellows.

Michael Anderson gave a comprehensive survey on Einstein metrics on open manifolds, which have a certain structure at infinity. Asymptotically, the simplest structures are those of constant curvature, and his five lectures covered both the asymptotically flat and the hyperbolic (or conformally-compact) cases. He described a large class of examples of such metrics, and then focused on general questions of existence and uniqueness. He explained through his talks the so-called Anti-de Sitter/Conformal Field Theory correspondence in physics, and gave a number of results on the structure of the map from the space of Einstein conformally-compact metrics to the conformal structures on the boundary. These lectures were masterful in their grasp of both the physical theories and the rigorous mathematical results related to them.

Karsten Grove gave a four-lecture mini-course on comparison geometry, from a fairly new and original point of view. The comparison geometry has its roots in global Riemannian geometry, where it took off in the 1930's through the works of Hopf, Morse, Schoenberg, Myers and Synge. The real breakthrough came in the 1950's with the pioneering works of Rauch, Alexandrov, Toponogov and Bishop. Since then, the simple idea of comparing the geometry of an arbitrary Riemannian manifold with the geometry of constant curvature spaces has had a tremendous evolution. First in conjunction with Morse theory and convexity, then with Gromov-Hausdorff topology on spaces of Riemannian manifolds, and the geometry of singular spaces, and most recently in the presence of symmetries. All these aspects were beautifully presented through the lectures which had a great success.

What is special about 6, 7 and 8 dimensions? Why do we study Calabi-Yau threefolds, G_2 and Spin(7) manifolds? These were the questions addressed in Nigel Hitchin's introductory lectures to these special geometries of great interest in string theory. Based on the fundamental principle to look at the geometry of open orbits of Lie groups, he presented a truly elegant and original approach to the subject. The orthogonal groups SO (n, n) also appear in this setting through their spin representations, and this provided an entrance into the exciting world of generalized geometry. His lectures were beautifully orchestrated with many examples and constructions on moduli spaces.

The audience during this first week was quite eclectic, ranging from physicists to geometers to topologists. All three lecturers did wonderful tutorial work during the office hours they kindly provided to the participants. There were many fruitful discussions across boundaries, and these led in particular to an informal lecture by Michael Anderson on the celebrated recent work of Perelman, aiming at a complete resolution of Thurston's geometrisation conjecture.

The last two weeks of the program included 43 one-hour specialized talks, elaborating in particular on the subjects introduced during the first week.

Einstein metrics, positive sectional curvature, obstruction theory for positive scalar curvature were the main topics during the second week of the program.

Michael Anderson presented his new construction of compact Einstein metrics, by using a generalization of Thurston's theory of hyperbolic Dehn surgery and glueing techniques. His construction provides the first known families of 4-dimensional compact Einstein manifolds which are neither Kählerian nor locally symmetric. Christoph Böhm gave a comprehensive overview of general existence results for homogeneous Einstein metrics, including many new examples. Charles Boyer lectured on his recent works with K. Galicki and J. Kollár that produce an abundance of new Einstein metrics on compact manifolds of dimension $2n + 1$, including exotic spheres. Don Page explained how an Euclidean version of the general Kerr-de Sitter metric leads to the construction of amazingly explicit Einstein metrics on S^{n-2} -bundles over S^2 . David Calderbank lectured on his recent works with H. Pedersen and M. Singer that classify both locally and globally self-dual Einstein manifolds and orbifolds that admit an isometric action of a 2-torus. Andrew Dancer presented a classification for superpotentials that have been used to reduce the Einstein equations to subsystems. Olivier Biquard explained the relation between 3-dimensional CR geometry and Einstein geometry, and defined a new eta invariant for CR manifolds coming from this correspondence. David Duchemin presented his work on quaternionic-Kähler fillings of quaternionic-contact structures in dimension 7. Gordon Craig explained his glueing construction of Einstein fillings of infinitely many topological types of certain 3-dimensional hyperbolic manifolds.

Wolfgang Ziller related the existence of positive sectional curvature metrics on the total space of fiber bundles with the recent constructions of Einstein self-dual orbifolds, due to N. Hitchin, D. Calderbank and M. Singer. Burkhard Wilking then presented a classification of the positively curved compact manifolds of cohomogeneity one, a result that he, K. Grove and W. Ziller have recently obtained, and he discussed in detail the still open case of a smooth 3-Sasakian 7-manifold constructed out of Hitchin's Einstein self-dual orbifolds. Ingi Petursson and Ailana Fraser talked about their exciting results on compact Riemannian manifolds of (almost) positive isotropic curvature.

Joachim Lohkamp presented the complete proof of the positive mass theorem that he and U. Christ have recently found. Marc Herzlich then defined an analogous asymptotic invariant in the case of asymptotically hyperbolic metrics, called "hyperbolic mass", and proved the "positive mass property" in this case. Margarita Kraus talked about inequalities which bound the Riemannian curvature tensor with the asymptotic "mass invariant" of an asymptotically flat Lorentzian manifold. Hélène Davaux found a new upper estimate of the scalar curvature of a compact spin manifold in terms of the spectrum of the Laplacian of the universal covering, thus providing a new obstruction to the existence of a positive constant scalar curvature metrics.

There were also two talks on upper curvature bounds during this second week, given by Regina Rotman and Krishnan Shankar. Alexandre Nabutovsky gave an excellent overview of variational methods in Riemannian Geometry, and Niky Kamran surveyed some long time behaviour results for solutions of geometric hyperbolic equations arising in the Lorenzian Kerr space-time.

The third week of the program was mostly focused around the geometry of metrics with special holonomy, Kähler and Hermitian metrics with special curvature, and holomorphic methods in Riemannian geometry.

Roger Bielawski lectured on his recent works on invariant Kähler metrics with prescribed Ricci curvature, on the complexification of a symmetric space of compact type. Paul Gauduchon gave a new description of all Kähler metrics with vanishing Bochner curvature, thus providing an alternative approach to R. Bryant's classification of these manifolds. Gideon Maschler presented his recent works with A. Derdzinski that classify, both locally and globally, all Einstein manifolds which are conformally Kähler. Christina Tønnesen-Friedman presented an abundance of new explicit examples of extremal Kähler metrics on toric bundles. Yann Rollin talked about his very important recent work with M. Singer that relates existence of scalar-flat Kähler metrics and stability of parabolic vector bundles. Maurizio Parton and Ruxandra Moraru talked about reduction and instanton moduli spaces over certain hyper-Hermitian manifolds.

Gueo Grantcharov, Ana Fino and Helge Joergensen explained their classification results of certain homogeneous spaces with special holonomy properties. Bogdan Alexandrov discussed some fuzzy threads in the literature concerning the notion of weak holonomy. Stefan Ivanov presented a proof of a Goldberg-type conjecture concerning Einstein G_2-manifolds. Georges Papadopoulos introduced the notion of spinorial cohomology and explained how it can be applied to study manifolds with special holonomy. Min-Oo talked about calibrated geometry in spaces with special holonomy G_2 and Spin(7), and its relevance to string theory. He discussed various explicit constructions recently found by physicists. His lecture was later followed by a talk by his collaborator, Spiro Karigiannis, on calibrated cycles in certain bundle constructions of metrics with holonomy G_2. There were two other talks on calibration theory associated to special Lagrangian manifolds, given by Marianty Ionel and Adrian Butscher. Wei-Dong Ruan discussed convergence and degeneration of complete Kähler-Einstein hypersurfaces in complex tori.

Robert Bryant lectured on complete Riemannian metrics for which the Ricci tensor is a Hessian of a function. This class of manifolds naturally appears in the study of Ricci flow on manifolds.

Claude LeBrun gave a very beautiful new proof of all the classical results concerning compact surfaces with closed geodesics, by using a twistor-theoretic approach to reduce these problems to certain rigidity properties of complex-analytic surfaces. Maciej Dunajski demonstrated how the twistor-approach can be applied to various non-linear integrable equations arising in mathematical physics to find solutions via simple algebro-geometric operations on families of rational curves.

Following Hitchin's ideas, Marco Gualtieri introduced the notion of generalized Kähler geometry and explained how these structures naturally appear on 4-manifolds and in connection with twisted K-theory classes of even dimensional Lie groups.

Thus, in three very intensive weeks, the program succeeded in tying together most of the new results in the subject and a variety of new projects were born. The participants affirmed frequently and spontaneously that the program was a great success.

The lecture notes of the short courses given during the first week of the program, as well as a number of other contributions will be published as a joint CRM-AMS volume.

Report on Stochastic 2004 - *continued from page 6*

Stochastic Networks Conference

Org.: Peter Glynn

This meeting attracted the major figures who are contributing to the development of a mathematical theory for stochastic networks. A large portion of the talks related to various challenges that are arising in the wireless networks setting, where key issues include the spatial variation, bandwidth limitations, and the need to generate distributed control policies. The format of the conference, in which only four one hour talks were scheduled over each of the six days (with the exception of the Wednesday session, in which only three talks were scheduled), lent itself to an environment in which participants were able to pursue significant technical interactions. In spite of the lack of opportunity to give full contributed talks, the meeting attracted a significant number of attendees, in large part because it provided an opportunity for unhurried technical exchanges amongst all participants.

It should be noted that Alcatel provided financial support to the meeting, in recognition of the relevance of the research agenda to its core mission. A number of industrial participants attended the meeting. In addition, one of the major journals in the area, *Queueing Systems: Theory and Applications,* contacted the Organizer subsequent to the meeting to request that a special invited issue, consisting of papers based on the talks given at the meeting, be put together. This special issue is being pursued, and publication is expected late in 2005. This journal's interest in pursuing such a special issue is a clear testimony to the quality of the talks offered as part of the Stochastic Networks Conference.

"A new feature for the stochastic networks conference was the inclusion of a couple of talks related to stochastic models in biology (I am thinking of the Kurtz and Hajek talks). I heard others also say that they found these talks very interesting for the fact that there was interesting mathematics related to models of interest to biologists".

A participant

Stochastic Networks Conference

Call Center Workshop Report

Org.: Raj Srinivasan

To recognize the recent flurry of research activities in the call center area, the last day of the SNC was designated as a joint activity with the CCW. There were four talks on that day of the SNC, most of them being concentrated on staffing large call centres using fluid models. This joint activity was very well received by the eighty-two people in attendance. The CCW continued for another two days with twelve talks. There were twenty-eight to forty-two people in attendance for the two-day activity. The talks covered a wide range of topics including queueing network with time varying rates, statistical analysis of call centres, workforce planning and profit maximization, revenue management through cross-selling and managing learning and turnover in employee staffing. The workshop also included a tutorial on call centers.

One of the highlight of the talks was the presentation from Chantal Gagné, the General Manager of Bell Canada Holdings, who discussed several challenges involved in the management of call centers at Bell Canada. In particular, she emphasized the difficulties facing the industry in forecasting call volume over the eighteen months planning horizon and training work force during that period. Active participation from the audience resulted in lively and productive discussions. Besides the annual two-day Call Center Forum run by the Wharton Financial Institutions Centre for their industrial participants, the CCW hosted by CRM is the first major workshop on call centres. Based on the feedback we have received from the participants, this workshop was a very successful one.

Le Bulletin du CRM

Volume 11, nº 1

Automne 2005

Le Bulletin du CRM est une lettre d'information à contenu scientifique, faisant le point sur les actualités du Centre de recherches mathématiques.
ISSN 1492 – 7659

Le Centre de recherches mathématiques (CRM) de l'Université de Montréal a vu le jour en 1969. Présentement dirigé par le professeur François Lalonde, il a pour objectif de servir de centre national pour la recherche fondamentale en mathématiques et dans leurs applications. Le personnel scientifique du CRM regroupe plus d'une centaine de membres réguliers et de boursiers post-doctoraux. De plus, le CRM accueille d'année en année un grand nombre de chercheurs invités.

Le CRM coordonne des cours gradués et joue un rôle prépondérant en collaboration avec l'ISM dans la formation de jeunes chercheurs. On retrouve partout dans le monde de nombreux chercheurs ayant eu l'occasion de perfectionner leur formation en recherche au CRM. Le Centre est un lieu privilégié de rencontres où tous les membres bénéficient de nombreux échanges et collaborations scientifiques.

Le CRM tient à remercier ses divers partenaires pour leur appui financier de notre mission: le Conseil de recherches en sciences naturelles et en génie du Canada, le Fonds québécois de la recherche sur la nature et les technologies, l'Université de Montréal, l'Université McGill, l'Université du Québec à Montréal, l'Université Concordia, l'Université Laval, l'Université d'Ottawa, ainsi que les fonds de dotation André-Aisenstadt et Serge-Bissonnette.

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Activités des laboratoires...

Montreal Scientific Computing Days

by Tony Humphries
26-27 February 2005, CRM
Speakers: Jay Gopalakrishnan (Florida), Jan S Hesthaven (Brown)
Organisers: Tony Humphries (McGill), Nilima Nigam (McGill), Robert Owens (Montréal).

The second edition of this annual event was once again a great success, this year attracting 101 participants, mostly from Québec and Ontario, up from 78 participants last year. The objective of this conference is to encourage scientific exchange within the scientific computing community in Québec and further afield. The two day program included two short courses offered by international experts, at a level accessible to advanced graduate students, as well as oral and poster contributed sessions. Students and post-doctoral researchers were especially encouraged to participate.

Jay Gopalakrishnan gave two long lectures on Multigrid Methods, a class of numerical techniques to solve linear systems arising from discretization of PDEs using a hierarchy of discretization grids. These methods can often compute an approximate solution up to a given precision at asymptotically optimal computational cost. The optimal complexity of multigrid has brought within the reach of simulation many scientific problems previously thought to be of intractable size. The first lecture covered the fundamental theory and mechanics of multigrid methods, showing how it leads to optimal algorithms, illustrated it on simple examples. The second lecture considered more complicated applications, highlighting examples from electromagnetics, and investigated the modifications needed to successfully apply the multigrid paradigm. The interplay between convergence theory of discretizations and convergence analysis of multigrid was empasized, especially how one has led to improvements in the other.

Jan Hesthaven gave two long lectures on Discontinuous Galerkin Methods for Solving Time-Dependent PDEs. These discontinuous finite element methods, although proposed first more than 3 decades ago, have recently received considerable attention due to a number of very attractive properties, e.g., solid theoretical foundation, ability to work with high-order and adaptive grids, support for unstructured grids and very high performance on parallel computers. The first lecture covered the fundamental theory of these methods, discussing some key theoretical results, illustrated by illuminating examples. The second lecture focused on more applied aspects and how to develop and implement these methods for a variety of problems and applications.

Both lecturers gave superb and well appreciated presentations, and also made comprehensive notes available. Hesthaven even supplied a suite of software routines for implementing the Discontinuous Galerkin Method. There were also 15 twenty minute contributed talks, many by postdoctoral fellows and graduate students, and a pizza and poster session on Saturday evening. The third Montreal Scientific Computing Days are scheduled for 25th and 26th February 2006; we look forward to seeing you there!

Conférence sur la théorie des points fixes et ses applications, en l'honneur du 75e anniversaire du Professeur Andrzej Granas

par Marlène Frigon

La Conférence sur la théorie des points fixes et ses applications a réuni à Montréal du 16 au 20 août 2004 une cinquantaine de participants venus célébrer dans une atmosphère de collégialité et de fraternité le 75e anniversaire du Professeur Andrzej Granas. Les thèmes principaux de la Conférence étaient la théorie des points fixes et ses applications à l'analyse non linéaire, aux équations différentielles et aux systèmes dynamiques. Notamment, les sujets suivants ont été traités : la théorie du degré topologique sur des espaces de Banach ou sur des variétés et la théorie du degré équivariant, les théorèmes de point fixe et de point périodique, la théorie de Nielsen et celle de Perron-Frobenius, les applications KKM et les théorèmes de type minimax, les méthodes variationnelles appliquées à des problèmes d'équations aux dérivées partielles.

Inférence Bayesienne en imagerie fonctionnelle cérébrale

par Jean-Marc Lina

Organisé conjointement par les Laboratoires PhysNum (J.-M. Lina) et de Statistique (J.-F. Angers), l'atelier 'Inférence Bayesienne en imagerie fonctionnelle cérébrale' (du 31 mars au 2 avril 2005) a attiré une trentaine de participants, la plupart provenant du secteur des mathématiques appliquées. En choisissant d'ancrer l'analyse Bayesienne dans le contexte de l'imagerie en neuroscience, les organisateurs avaient comme objectif de susciter l'intérêt des jeunes chercheurs quant aux avenues de recherche qui s'offrent aujourd'hui dans ce domaine extrêmement dynamique des biosciences. Charles Bouman (Purdue Univ, USA) a fait le point sur les nouveaux imageurs optiques qui soulèvent aujourd'hui autant d'intérêt sur le plan des possibilités de 'voir' l'activité cérébrale que de défis au niveau des méthodologies de reconstruction tomographique. La tomographie Bayesienne a fait l'objet de l'exposé d'Ali Mohammad-Djafari (Supélec, France), spécialiste reconnu des questions de reconstruction à partir de données partielles et incomplètes. L'exposé de K. Worsley (McGill, Canada) donnait un point de vue critique sur l'approche Bayesienne en le remettant dans une perspective d'étude de groupe pour l'inférence robuste de l'activation cérébrale. Cet exposé complétait fort bien les deux présentations fouillées de C. Phillips (Université de Liège, Belgique) qui a su remettre en perspective les outils méthodologiques dans le contexte appliqué de l'imagerie cérébrale qui peut, aujourd'hui, profiter de la fusion de plusieurs types d'information. L'atelier a également donné l'occasion à de jeunes chercheurs de présenter leurs propres travaux. C. Grova (McGill) a ainsi présenté une méthodologie d'évaluation pour comparer les performances des estimateurs; J. Daunizeau (U. de Montréal) a donné une solution au problème de fusion multimodale qui permet de prendre en compte différentes sources d'information et F. Destrempes (U. de Montréal) a présenté un estimateur Bayesien pour les modèles de champs de Markov cachés qui sont largement utilisés en imagerie mathématique. L'ensemble des présentations est disponible sur la page www : http://www.crm.umontreal.ca/Bayes05/index.html

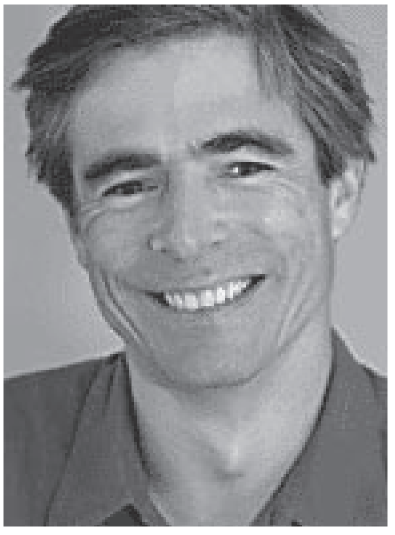
Message du directeur

Le CRM est un ensemble vibrant réunissant une centaine de professeurs, une cinquantaine de postdoctorants et un grand nombre d'étudiants qui appartiennent à ses huit laboratoires, chacun réparti dans sept universités au Québec et en Ontario. Avec sa programmation thématique dont le CRM a été l'un des pionniers dans le monde, avec son Colloque hebdomadaire CRM-ISM, son programme postdoctoral qui a attiré l'an dernier 350 candidatures de niveau exceptionnel provenant surtout des États-Unis, du Canada et d'Europe, avec ses activités qui englobent l'imagerie médicale, la bioinformatique, l'optimisation combinatoire, la chimie quantique et les méthodes numériques pour les systèmes différentiels de très grande dimension, le CRM reste un modèle de ce que la recherche mathématique pure et appliquée peut apporter de mieux.

Le site web du CRM, qui attire chaque semaine des milliers de visiteurs, a fait peau neuve. Il pourra désormais permettre aux centaines de candidats au programme postdoctoral CRM-ISM de postuler en ligne.

Cette année, le CRM lance son année thématique en Analyse et théorie des nombres, organisée par Henri Darmon, Chantal David et Andrew Granville. Avec un budget qui inclut des fonds importants de la NSF et du Clay Institute aussi bien que ceux des Chaires de recherche du Canada, le CRM renouera avec la tradition de recevoir des visiteurs pour une très longue période allant d'un semestre à une année. Un institut comme le CRM n'est pas qu'un centre de conférences, c'est aussi un lieu où la recherche avancée se fait, et cela prend souvent plus de temps que la durée habituelle d'un simple atelier.

Le CRM lancera dès l'an prochain une programmation thématique duale, selon le modèle suivant: dans le thème annuel choisi, l'un des deux semestres portera sur des questions de mathématiques pures alors que le second abordera les aspects plus appliqués. Le thème de l'année 2006-2007 sera la combinatoire: le premier semestre, plus appliqué et piloté avec beaucoup de maîtrise par Odile Marcotte, portera sur l'optimisation combinatoire avec des méthodes exactes et probabilistes – deux chaires Aisenstadt y participeront (Paul Seymour et Noga Alon, qui est, incidemment, le président du comité du programme scientifique du ICM 2006); le second semestre, piloté par François Bergeron et Mark Hayman, portera sur les aspects plus purs, de la géométrie algébrique à la physique statistique – j'ai également demandé à Slava Kharlamov et Rahul Pandharipande d'y organiser un atelier qui réunira pour la première fois des experts des questions énumératives en géométrie réelle et complexe (qui, étrangement, ne se rencontrent pas si souvent qu'on pourrait le croire).



François Lalonde

L'année 2007-2008 sera consacrée à la dynamique dans un sens très large, pure et appliquée. Même dans son sens “pur”, cette année comprendra des questions très variées de géométrie riemannienne, d'équations d'évolution intrinsèque (c'est-à-dire) dont l'inconnue est une donnée intrinsèque comme la métrique par exemple, comme c'est le cas du programme de Perelman, et extrinsèque (par exemple) évolution d'une hypersurface qui tend a minimiser une fonctionnelle donnée, des questions de dynamique hamiltonienne en topologie symplectique, et des questions plus classiques de dynamique auxquelles Sasha Shnirelman s'est attelé récemment. Il y a déjà un grand nombre de mathématiciens impliqués dans ce semestre, dont V. Apostolov, O. Cornea, P. Guan, D. Jakobson, I. Polterovich, A. Shnirelman, A. Stancu. Tony Humphries pilotera la partie plus appliquée de l'année.

François Lalonde